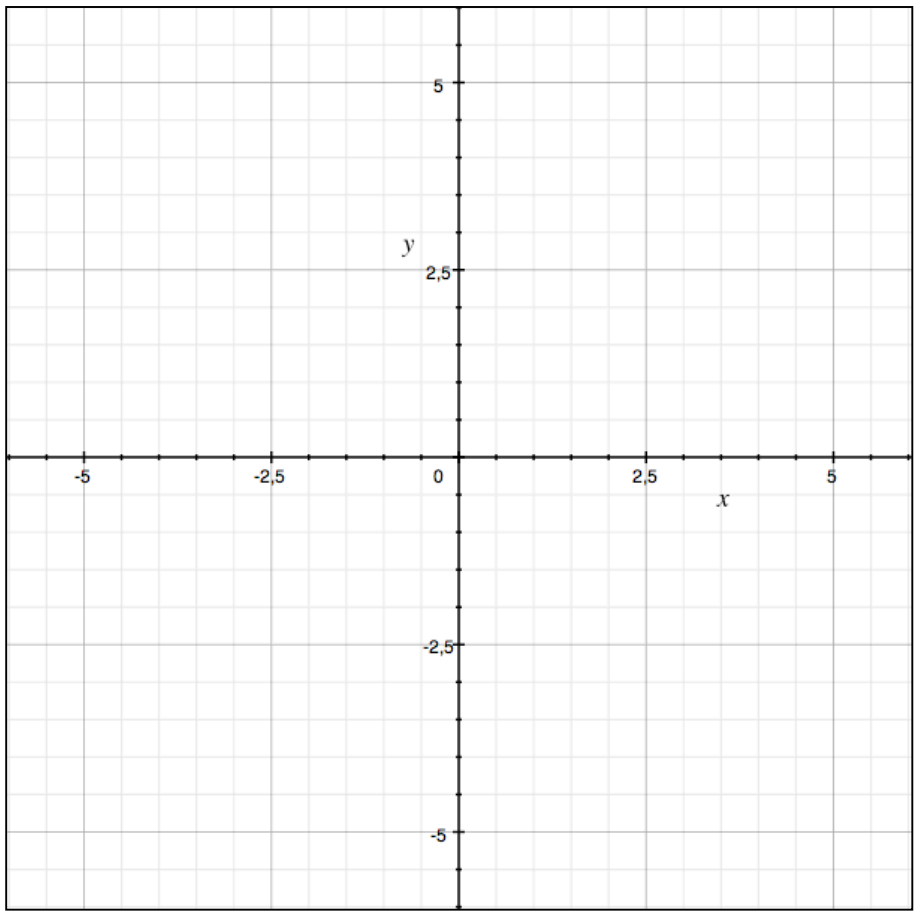
Review of second degree and Homographic functions

References : Memos available on the website of the class

I . 1. Draw the parabola and the 2 lines defined by the following equations :

$$(1)\quad y=-\frac{1}{4}x^2+\frac{1}{2}x+2\quad ;\quad (2)\quad y=-\frac{1}{2}x+2\quad ;\quad (3)\quad y=\frac{1}{2}x+2$$

I.2. Find the coordinates of the intersection points of the 2 lines with the Parabola.



I.3. Let m be any real number. We consider the Staight line $\mathbf{(D_m)}$ defined by the equation $\boldsymbol{y = mx + 2}$

a) Show that $\mathbf{(D_m)}$ turns around a fixed point A while m varies from $-\infty$ to $+\infty$.

b) Find for which values of m the line $\mathbf{(D_m)}$ cuts the Parabola in 2 points (write the proof).

c) Explain why the line $\mathbf{(D_{1/2})}$ is tangent to the Parabola in A(0;2).

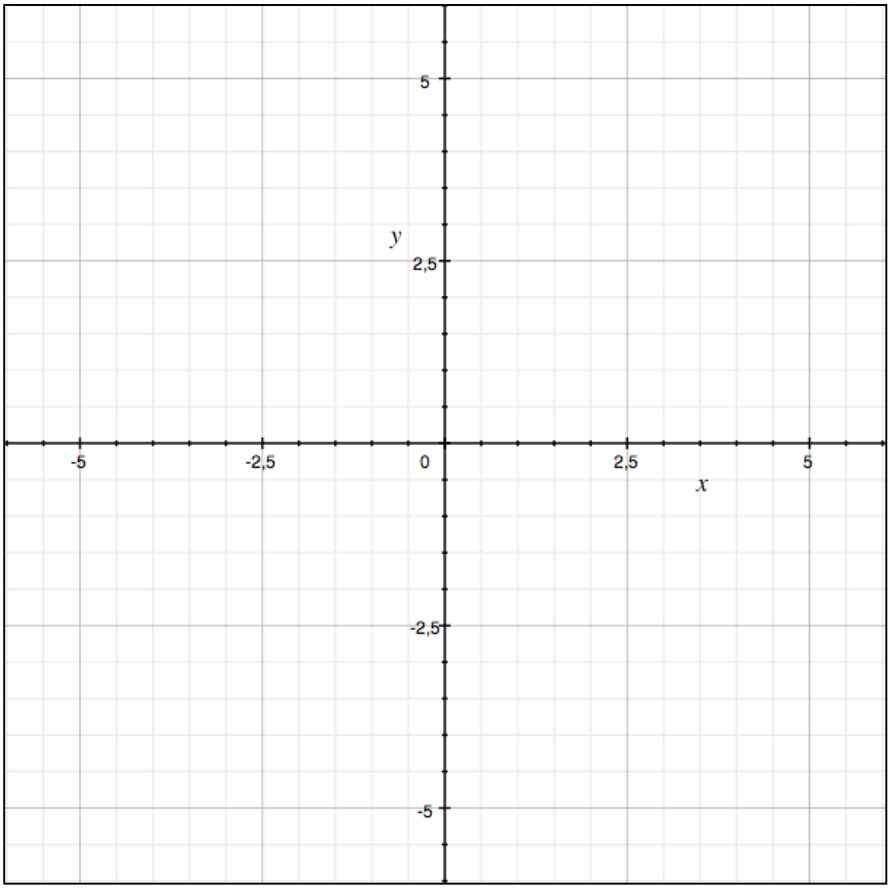
II. Lets consider the following functions :

$$f : x \mapsto y = \frac{1}{4}x^2 + \frac{1}{2}x - 2 \quad (P) \quad ; \quad h : x \mapsto y = \frac{2x+8}{x+1} \quad (H)$$

II.1 Draw the Parabola (P) and the hyperbola (H) in the same system of coordinates below.

II. 2 Calculate the coordinates of the intersection points of (P) and (H).

(show all your calculations here).



II.3. Shade the domain of the plane which are the solutions of the following system :

$$\begin{cases} x \leq -1 \\ y \geq \frac{1}{4}x^2 + \frac{1}{2}x - 2 \\ y \leq \frac{2x+8}{x+1} \end{cases}$$