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Please use scrap paper, or back of the page, before copying your answers in the spaces below.

Problem I :

$$f(x)=\frac{x^3+2x^2}{2(x-1)^2}$$

1. Break $f(x)$ into the form $ax+b+\dfrac{cx+d}{2(x-1)^2}$ $(x\neq 1)$ .... 4 pts

2. Find the limits of f at the ends of each interval of its definition set .... 4 pts
   [Use back of page to justify answers]

3. Show that (C_f) has a “vertical” and an oblique asymptote (Δ), give their equations. .... 4 pts

4. Justify the position of (C_f) with respect to this asymptote. .... 2 pts

5. Find the derivative $f\,'(x)$ and factor it in binomials .... 6 pts

6. Give the zeroes of $f\,'(x)$ and justify the signs of the derivative. .... 6 pts

7. Find the intersections of (C_f) with the axes of coordinates. .... 4 pts

8. Find the intersection of (C_f) with its asymptote (Δ). .... 2 pts

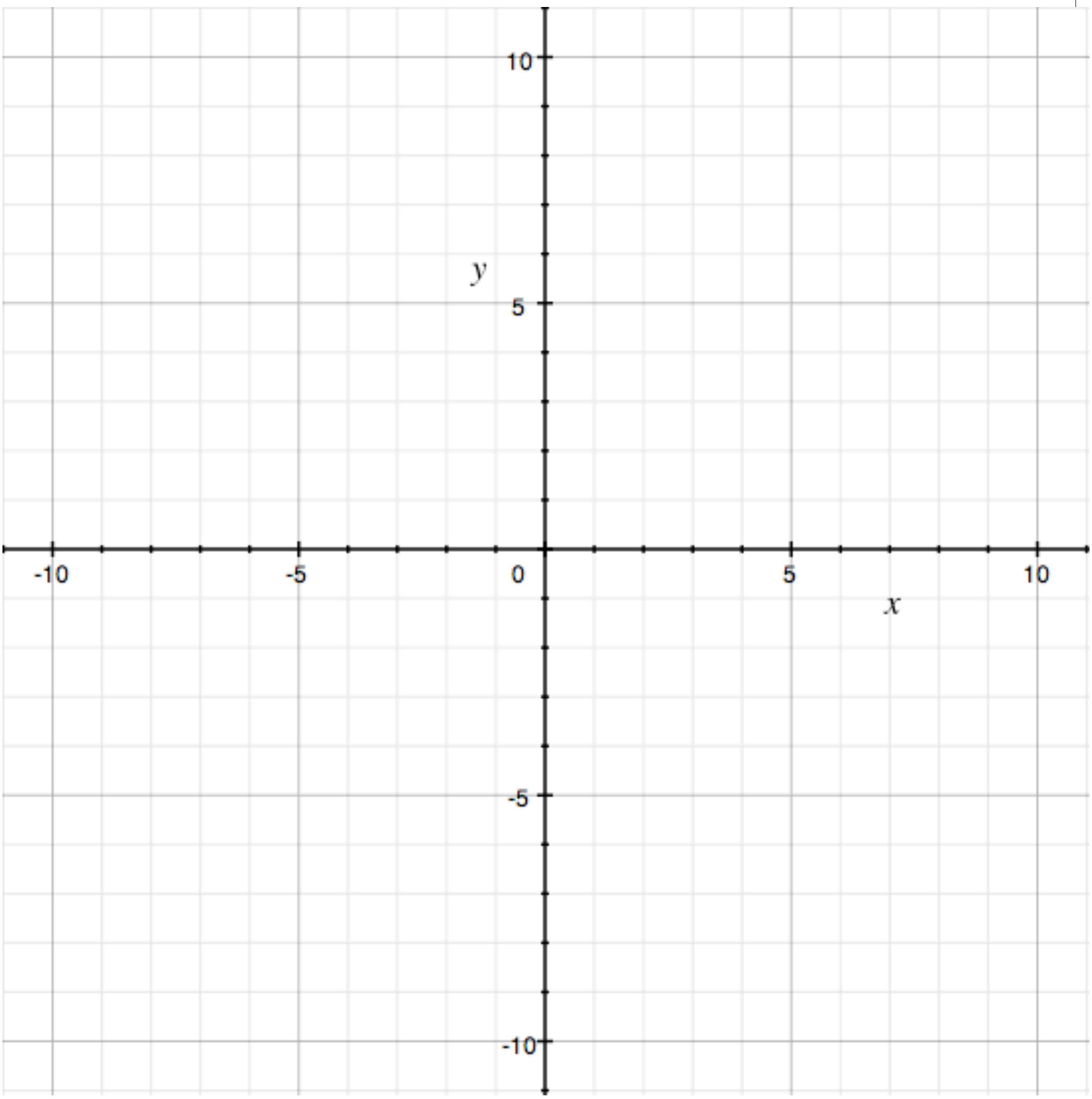
9. Find the equation of the tangent line in $A(-2;0)$ to (C_f) .... 2 pts

10. Show <u>all</u> the previous results in the chart below. .... 6 pts

x	$-\infty$
Sign [f '(x)]	
Variations *and* ***limits*** of f	

11. Give the approximate decimal value of the extremes here : .... 2 pts

12. Draw carefully (C_f) its asymptotes, and the tangent line (T_A). .... 8 pts



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Problem II :

$$f(x)=\sqrt{\left|\frac{x^3+x^2-2}{x+2}\right|}$$

1. Break the function f in two different functions f_1 and f_2 without absolute values, and specify their intervals of definition : .... 4 pts

   $f_1(x) =$ $D_{f1} =$; $f_2(x) =$ $D_{f2} =$

2. Find the limits of f at the ends of each interval. .... 4 pts
   [Use back of page to fully justify answers]

3. Show that (C_f) has two oblique asymptotes $(\Delta_1)\ y = x - \frac{1}{2}$ & $(\Delta_2)\ y = -x + \frac{1}{2}$ .... 4 pts
   [Use back of page to fully justify answers]

4. Justify the position of (C_f) with respect to these two asymptotes .... 2 pts

5. *Give* the derivatives of the functions f_1 and f_2 on each interval where they are defined.......... 8 pts

6. Show that the sign of the derivatives depends on $P(x) = 2x^3 + 7x^2 + 4x + 2$ .... 6 pts
   and show that there is only one zero α for P(x), with $-3.0 < \alpha < -2.9$ *[Use back of page]*

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7. Study the singularity of (C_f) in $A(1;0)$ by studying the limits of the rate of growth of the function f...... 6 pts
on both sides of A.

8. Show the previous results in the chart below, with the intersections of (C_f) with the axes. .... 6 pts

x	$-\infty$ $+\infty$
Sign of $[f_1\,'(x)]$	
Sign of $[f_2\,'(x)]$	
Sign of $[f\,'(x)]$	
Variations *and* ***limits*** of f	

9. Give the approximate decimal value of the extremes here.: .... 2 pts

10. Draw carefully (C_f) its asymptotes, and the tangent line (T_A). .... 8 pts