Definition & Construction of a Parabola (Part 1)

Let f be the function defined by : $\boxed{f : x \mapsto ax^2}$ (a≠0)

I- **Algebraic properties** :

1°) ***Even*** *function* : for any $x \in \mathbb{R}$, $\boxed{f(-x) = f(x)}$.

2°) Rate of growth ***non constant*** : $T_{[f,(x_1;x_2)]} = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = a(x_2 + x_1)$

3°) Sign of T = Sign of a on [0 ; +∞[, and Sign of T = Sign of (-a) on]- ∞ ; 0]

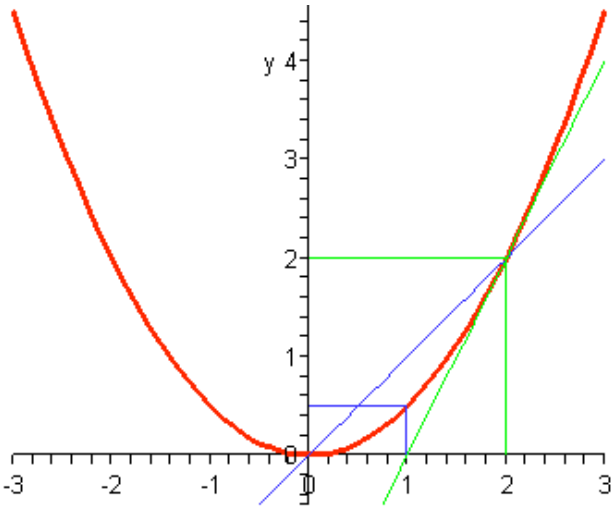
4°) Chart of the Variations of f :

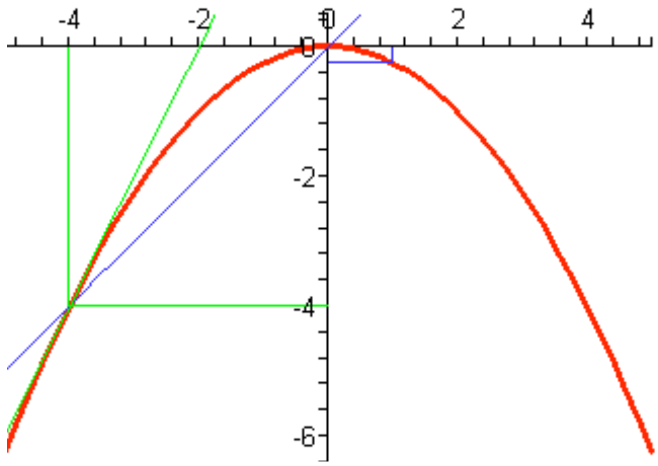
a > 0					
x	-∞	-1	0	1	+∞
T		-	‖	+	
f	+∞	a ↘	0	a ↗	+∞

a < 0					
x	-∞	-1	0	1	+∞
T		+	‖	-	
f	- ∞	a ↗	0	a ↘	- ∞

II- **Geometric Properties** :

1°) The curve has (Oy) as an axis of symmetry. For that reason the curve is called a **Parabola.**
2°) The Parabola is tangent to the (Ox) Axis in O.
3°) The Parabola passes through the point A(1 ; a).
4°) If a > 0 the Parabola concavity is directed towars the positive y :
*(*as one can say the *« the bowl can hold water »)*
If a < 0 the Parabola concavity is directed towards the négative y :
*(*as one can say the *« the bowl cannot hold water »)*
5°) The Parabola intercepts the 1st bisector line (y = x) at point B(1/a ; 1/a)
6°) On B the Parabola is tangent to the line joining B to the middle of the segment located under the tangent, which is the point of abscissa 1/2a
7°) By symmetry with respect to Oy we get the points A'(-1 ;a) and B'(-1/a ; 1/a)
8°) If a is very small compared to the unity (a << 1) , the parabola is widely opened, inversely if a >>1 the parabola is very narrow around the axis of symmetry.
9°) The parabola contains absolutely no piece of a straight line.
10°) The branches spread indefinitely in the direction of the Oy axis.





Second degree functions (Part 2)

Second degree functions are in the general form : $f : x \mapsto ax^2 + bx + c$ *with* $a \neq 0$
This expression can take any of the following forms :

(P_1)	$y = a\,x^2$	
(P_2)	$y = a\,x^2 + H$	
(P_3)	$y = a\,(x - L)^2$	
(P_4)	$y = a\,(x - L)^2 + H$	
(P_5)	$y = a\,(x - x')(x - x'')$	
(P_6)	$y = a\,x^2 + bx + c$	*(trinomial)*

1°) Transformation from (P_1) to (P_2) is a **Translation** defined by the vertical vector $H\vec{j}$ (parallel to the (Oy) axis. (P_2) intercepts (Oy) in y = H. *(H = « Hight » ; L = « Length »)*

2°) Transformation from (P_1) to (P_3) is a **Translation** defined by the horizontal vector $L.\vec{i}$ (parallel to the (O*x*) axis)

3°) Transformation from (P_1) to (P_4) is a **Translation** of vector $\vec{V} = L.\vec{i} + H.\vec{j}$
The Parabola (P_4) has a vertex in O'(L ;H).
Let **X = *x* – L** and **Y = *y* – H** then **Y = a X²** which means that (P_4) is Symmetrical whith respect of the axis defined by x = L (parallel to (Oy))
(P_4) is drawn in the system (O'X,O'Y) just like (P_1) in the system (*Ox*,*Oy*).

4°) The Parabola (P_5) intercepts the axis (Ox) in x' and x'', its vertex is then at

S of abscissa = $\frac{x' + x''}{2} = \frac{b}{2a}$ ordinate H = *f*(L).

5°) To build the parabola (P_6) one can either :
a. use the form (P_4) by breaking the trinomial in that « canonic » form.
b. find the coordinates of the vertex **O'{L=- b/2a ; H=*f*(L) }** then find the Ox and Oy intersection pts **:** on (Oy) : (x = 0 ; y = c) et (Ox) solutions of the équation $a\,x^2 + bx + c = 0$ *(if any)*.

Example *: let (P) be the Parabola defined by* $y = 1/4\,(x - 2)^2 + 3$ then L=2 ; H=3 ; Y = ¼ X²