Leonhard Paul Euler (15 April 1707 – 18 September 1783) was a pioneering Swiss mathematician and physicist who spent most of his life in Russia and Germany. His surname is pronounced /ˈɔɪlər/ *OY-lər* in English and [ˈɔʏlɐ] in German; the common English pronunciation /ˈjuːlər/ *EW-lər* is incorrect.

Euler made important discoveries in fields as diverse as infinitesimal calculus and graph theory. He also introduced much of the modern mathematical terminology and notation, particularly for mathematical analysis, such as the notion of a mathematical function.[5] He is also renowned for his work in mechanics, fluid dynamics, optics, and astronomy.

Euler is considered to be the preeminent mathematician of the 18th century and one of the greatest of all time. He is also one of the most prolific; his collected works fill 60–80 quarto volumes.[6] A statement attributed to Pierre-Simon Laplace expresses Euler's influence on mathematics: "Read Euler, read Euler, he is **the master of us all**."[7]

Euler was featured on the sixth series of the Swiss 10-franc banknote and on numerous Swiss, German, and Russian postage stamps. The asteroid 2002 Euler was named in his honor. He is also commemorated by the Lutheran Church on their Calendar of Saints on 24 May – he was a devout Christian (and believer in biblical inerrancy) who wrote apologetics and argued forcefully against the prominent atheists of his time.[8

Early years

Euler was born in Basel to Paul Euler, a pastor of the Reformed Church, and Marguerite Brucker, a pastor's daughter. He had two younger sisters named Anna Maria and Maria Magdalena. Soon after the birth of Leonhard, the Eulers moved from Basel to the town of Riehen, where Euler spent most of his childhood. Paul Euler was a friend of the Bernoulli family—Johann Bernoulli, who was then regarded as Europe's foremost mathematician, would eventually be the most important influence on young Leonhard. Euler's early formal education started in Basel, where he was sent to live with his maternal grandmother. At the age of thirteen he enrolled at the University of Basel, and in 1723, received his M.Phil. with a dissertation that compared the philosophies of Descartes and Newton. At this time, he was receiving Saturday afternoon lessons from Johann Bernoulli, who quickly discovered his new pupil's incredible talent for mathematics.[9] Euler was at this point studying theology, Greek, and Hebrew at his father's urging, in order to become a pastor, but Bernoulli convinced Paul Euler that Leonhard was destined to become a great mathematician. In 1726, Euler completed his Ph.D. dissertation on the propagation of sound with the title *De Sono*[10] and in 1727, he entered the *Paris Academy Prize Problem* competition, where the problem that year was to find the best way to place the masts on a ship. He won second place, losing only to Pierre Bouguer—who is now known as "the father of naval architecture". Euler subsequently won this coveted annual prize twelve times in his career.[11]

St. Petersburg

Around this time Johann Bernoulli's two sons, Daniel and Nicolas, were working at the Imperial Russian Academy of Sciences in St Petersburg. In July 1726, Nicolas died of appendicitis after spending a year in Russia, and when Daniel assumed his brother's position in the mathematics/physics division, he recommended that the post in physiology that he had vacated be filled by his friend Euler. In November 1726 Euler eagerly accepted the offer, but delayed making the trip to St Petersburg while he unsuccessfully applied for a physics professorship at the University of Basel.[12]

1957 stamp of the former Soviet Union commemorating the 250th birthday of Euler. The text says: 250 years from the birth of the great mathematician, academician Leonhard Euler.

Euler arrived in the Russian capital on 17 May 1727. He was promoted from his junior post in the medical department of the academy to a position in the mathematics department. He lodged with Daniel Bernoulli with whom he often worked in close collaboration. Euler mastered Russian and settled into life in St Petersburg. He also took on an additional job as a medic in the Russian Navy.[13]

The Academy at St. Petersburg, established by Peter the Great, was intended to improve education in Russia and to close the scientific gap with Western Europe. As a result, it was made especially attractive to foreign scholars like Euler. The academy possessed ample financial resources and a comprehensive library drawn from the private libraries of Peter himself and of the nobility. Very few students were enrolled in the academy so as to lessen the faculty's teaching burden, and the academy emphasized research and offered to its faculty both the time and the freedom to pursue scientific questions.[11]

The Academy's benefactress, Catherine I, who had continued the progressive policies of her late husband, died on the day of Euler's arrival. The Russian nobility then gained power upon the ascension of the twelve-year-old Peter II. The nobility were suspicious of the academy's foreign scientists, and thus cut funding and caused other difficulties for Euler and his colleagues.

Conditions improved slightly upon the death of Peter II, and Euler swiftly rose through the ranks in the academy and was made professor of physics in 1731. Two years later, Daniel Bernoulli, who was fed up with the censorship and hostility he faced at St. Petersburg, left for Basel. Euler succeeded him as the head of the mathematics department.[14]

On 7 January 1734, he married Katharina Gsell (1707–1773), a daughter of Georg Gsell, a painter from the Academy Gymnasium.[15] The young couple bought a house by the Neva River. Of their thirteen children, only five survived childhood.[16]

Berlin

Stamp of the former German Democratic Republic honoring Euler on the 200th anniversary of his death. In the middle, it shows his polyhedral formula $V + F - E = 2$.

Concerned about the continuing turmoil in Russia, Euler left St. Petersburg on 19 June 1741 to take up a post at the *Berlin Academy*, which he had been offered by Frederick the Great of Prussia. He lived for twenty-five years in Berlin, where he wrote over 380 articles. In Berlin, he published the two works which he would be most renowned for: the *Introductio in analysin infinitorum*, a text on functions published in 1748, and the *Institutiones calculi differentialis*,[17] published in 1755 on differential calculus.[18] In 1755, he was elected a foreign member of the Royal Swedish Academy of Sciences.

In addition, Euler was asked to tutor the Princess of Anhalt-Dessau, Frederick's niece. Euler wrote over 200 letters to her, which were later compiled into a best-selling volume entitled *Letters of Euler on different Subjects in Natural Philosophy Addressed to a German Princess*. This work contained Euler's exposition on various subjects pertaining to physics and mathematics, as well as offering valuable insights into Euler's personality and religious beliefs. This book became more widely read than any of his mathematical works, and it was published across Europe and in the United States. **The popularity of the 'Letters' testifies to Euler's ability to communicate scientific matters effectively to a lay audience, a rare ability for a dedicated research scientist.[18]**

Despite Euler's immense contribution to the Academy's prestige, he was eventually forced to leave Berlin. This was partly because of a conflict of personality with Frederick, who came to regard Euler as unsophisticated, especially in comparison to the circle of philosophers the German king brought to the Academy. Voltaire was among those in Frederick's employ, and the Frenchman enjoyed a prominent position in the king's social circle. Euler, a simple religious man and a hard worker, was very conventional in his beliefs and tastes. He was in many ways the direct opposite of Voltaire. Euler had limited training in rhetoric, and tended to debate matters that he knew little about, making him a frequent target of Voltaire's wit.[18]

Frederick also expressed disappointment with Euler's practical engineering abilities:

A 1753 portrait by Emanuel Handmann. This portrayal suggests problems of the right eyelid, and possible strabismus. The left eye appears healthy; it was later affected by a cataract.[20]

Eyesight deterioration

Euler's eyesight worsened throughout his mathematical career. Three years after suffering a near-fatal fever in 1735 he became nearly blind in his right eye, but Euler rather blamed his condition on the painstaking work on cartography he performed for the St. Petersburg Academy. Euler's sight in that eye worsened throughout his stay in Germany, so much so that Frederick referred to him as "Cyclops". Euler later suffered a cataract in his good left eye, rendering him almost totally blind a few weeks after its discovery in 1766. Even so, his condition appeared to have little effect on his productivity, as he compensated for it with his mental calculation skills and photographic memory. For example, Euler could repeat the Aeneid of Virgil from beginning to end without hesitation, and for every page in the edition he could indicate which line was the first and which the last. With the aid of his scribes, Euler's productivity on many areas of study actually increased. He produced on average one mathematical paper every week in the year 1775.[6]

Return to Russia

The situation in Russia had improved greatly since the accession to the throne of Catherine the Great, and in 1766 Euler accepted an invitation to return to the St. Petersburg Academy and spent the rest of his life in Russia. His second stay in the country was marred by tragedy. A fire in St. Petersburg in 1771 cost him his home, and almost his life. In 1773, he lost his wife of 40 years. Three years after his wife's death Euler married her half sister, Salome Abigail Gsell (1723–1794).[21] This marriage would last until his death.

On 18 September 1783, Euler died in St. Petersburg after suffering a brain hemorrhage, and was buried with his wife in the Smolensk Lutheran Cemetery on Vasilievsky Island (the Soviets destroyed the cemetery after transferring Euler's remains to the Orthodox Alexander Nevsky Lavra). His eulogy was written for the French Academy by the French mathematician and philosopher Marquis de Condorcet, and an account of his life, with a list of his works, by Nikolaus von Fuss, Euler's son-in-law and the secretary of the Imperial Academy of St. Petersburg. Condorcet commented : *...il cessa de calculer et de vivre.*

Contributions of Euler to mathematics and physics

Euler worked in almost all areas of mathematics: geometry, infinitesimal calculus, trigonometry, algebra, and number theory, as well as continuum physics, lunar theory and other areas of physics. He is a seminal figure in the history of mathematics; if printed, his works, many of which are of fundamental interest, would occupy between 60 and 80 quarto volumes.[6] Euler's name is associated with a large number of topics.

Mathematical notation

Euler introduced and popularized several notational conventions through his numerous and widely circulated textbooks. Most notably, he introduced the concept of a function[5] and was the first to write f(x) to denote the function f applied to the argument x. He also introduced the modern notation for the trigonometric functions, the letter e for the base of the natural logarithm (now also known as Euler's number), the Greek letter Σ for summations and the letter i to denote the imaginary unit.[23] The use of the Greek letter π to denote the ratio of a circle's circumference to its diameter was also popularized by Euler, although it did not originate with him.[24]

Analysis

The development of infinitesimal calculus was at the forefront of 18th century mathematical research, and the Bernoullis—family friends of Euler—were responsible for much of the early progress in the field. Thanks to their influence, ***studying calculus became the major focus of Euler's work****. While some of Euler's proofs are not acceptable by modern standards of mathematical rigour,[25] his ideas led to many great advances. Euler is well-known in analysis for his frequent use and development of* **power series***, the expression of functions as sums of infinitely many terms, such as*

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = \lim_{n\to\infty} \left(\frac{1}{0!} + \frac{x}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!}\right).$$

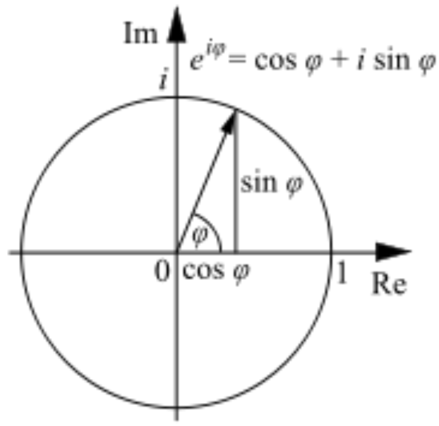
Notably, Euler directly proved the power series expansions for e and the inverse tangent function. (Indirect proof via the inverse power series technique was given by Newton and Leibniz between 1670 and 1680.) His daring (and, by modern standards, technically incorrect) use of power series enabled him to solve the famous Basel problem in 1735:[25]

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \lim_{n\to\infty} \left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2}\right) = \frac{\pi^2}{6}.$$

A geometric interpretation of Euler's formula

Euler introduced the use of the exponential function and logarithms in analytic proofs. He discovered ways to express various logarithmic functions using power series, and he successfully defined logarithms for negative and complex numbers, thus greatly expanding the scope of mathematical applications of logarithms.[23] He also defined the exponential function for complex numbers, and discovered its relation to the trigonometric functions. For any real number φ, Euler's formula states that the complex exponential function satisfies

$$e^{i\varphi} = \cos\varphi + i\sin\varphi.$$



A special case of the above formula is known as Euler's identity,

$$e^{i\pi} + 1 = 0$$

called **"the most remarkable formula in mathematics"** *by Richard Feynman, for its single uses of the notions of addition, multiplication, exponentiation, and equality, and the single uses of the important constants 0, 1, e, i and π.[26] In 1988, readers of the Mathematical Intelligencer voted it "the Most Beautiful Mathematical Formula Ever".[27] In total, Euler was responsible for three of the top five formulae in that poll.[27]*

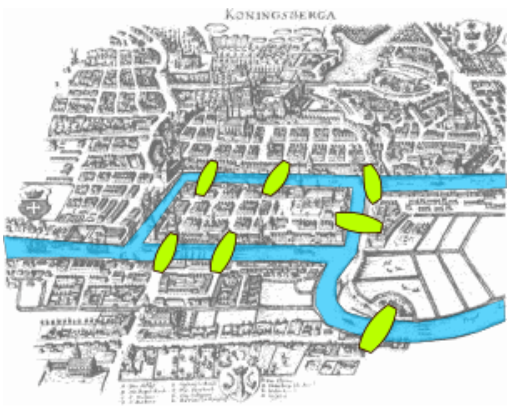
De Moivre's formula is a direct consequence of Euler's formula.
In addition, Euler elaborated the theory of higher transcendental functions by introducing the gamma function and introduced a new method for solving quartic equations. He also found a way to calculate integrals with complex limits, foreshadowing the development of modern complex analysis, and invented the calculus of variations including its best-known result, the Euler–Lagrange equation.
Euler also pioneered the use of analytic methods to solve number theory problems. In doing so, he united two disparate branches of mathematics and introduced a new field of study, analytic number theory. In breaking ground for this new field, Euler created the theory of hypergeometric series, q-series, hyperbolic trigonometric functions and the analytic theory of continued fractions. For example, he proved the infinitude of primes using the divergence of the harmonic series, and he used analytic methods to gain some understanding of the way prime numbers are distributed. Euler's work in this area led to the development of the prime number theorem.[28]

Number theory
Euler's interest in number theory can be traced to the influence of Christian Goldbach, his friend in the St. Petersburg Academy. A lot of Euler's early work on number theory was based on the works of Pierre de Fermat. Euler developed some of Fermat's ideas, and disproved some of his conjectures.
Euler linked the nature of prime distribution with ideas in analysis. He proved that the sum of the reciprocals of the primes diverges. In doing so, he discovered the connection between the Riemann **zeta function** *and the prime numbers; this is known as the Euler product formula for the Riemann zeta function.*
Euler proved Newton's identities, Fermat's little theorem, Fermat's theorem on sums of two squares, *and he made distinct contributions to Lagrange's four-square theorem. He also invented the* **totient function φ(n)** *which is the number of positive integers less than or equal to the integer n that are coprime to n. Using properties of this function, he generalized Fermat's little theorem to what is now known as Euler's theorem. He contributed significantly to the theory of perfect numbers, which had fascinated mathematicians since Euclid. Euler also made progress toward the prime number theorem, and he conjectured the law of quadratic reciprocity. The two concepts are regarded as fundamental theorems of number theory, and his ideas paved the way for the work of Carl Friedrich Gauss.*[29]
By 1772 Euler had proved that $2^{31} - 1 = 2{,}147{,}483{,}647$ *is a Mersenne prime. It may have remained the largest known prime until 1867.*[30]

Geometry

- *Euler's line*
- *Euler's circle*
- *Affine geometry*

Graph theory



Map of Königsberg in Euler's time showing the actual layout of the seven bridges, highlighting the river Pregel and the bridges.
In 1736, Euler solved the problem known as the Seven Bridges of Königsberg.[31] *The city of Königsberg, Prussia was set on the Pregel River, and included two large islands which were connected to each other and the mainland by seven bridges. The problem is to decide whether it is possible to follow a path that crosses each bridge exactly once and returns to the starting point. It is not: there is no Eulerian circuit. This solution is considered to be the first theorem of graph theory, specifically of planar graph theory.*[31]

Euler also discovered the formula V − E + F = 2
relating the number of vertices, edges, and faces of a convex polyhedron[32], and hence of a planar graph (for a planar graph, **V − E + F = 1***). The constant in this formula is now known as the Euler characteristic for the graph (or other mathematical object), and is related to the genus of the object.[33] The study and generalization of this formula, specifically by Cauchy[34] and L'Huillier,[35] is at the origin of topology.*

Applied mathematics
Some of Euler's greatest successes were in solving real-world problems analytically, and in describing numerous applications of the Bernoulli numbers, Fourier series, Venn diagrams, Euler numbers, the constants e and π, continued fractions and integrals. He integrated Leibniz's differential calculus with Newton's Method of Fluxions, and developed tools that made it easier to apply calculus to physical problems. He made great strides in improving the numerical approximation of integrals, inventing what are now known as the Euler approximations. The most notable of these approximations are Euler's method and the Euler–Maclaurin formula. He also facilitated the use of differential equations, in particular introducing the Euler–Mascheroni constant:

$$\gamma = \lim_{n\to\infty}\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{n}-\ln(n)\right).$$

One of Euler's more unusual interests was the application of **mathematical ideas in music**.
In 1739 he wrote the Tentamen novae theoriae musicae*, hoping to eventually incorporate musical theory as part of mathematics. This part of his work, however, did not receive wide attention and was once described as too mathematical for musicians and too musical for mathematicians.*

Physics and astronomy
Euler helped develop the Euler-Bernoulli beam equation, which became a cornerstone of engineering. Aside from successfully applying his analytic tools to problems in classical mechanics, Euler also applied these techniques to celestial problems. His work in astronomy was recognized by a number of Paris Academy Prizes over the course of his career. His accomplishments include determining with great accuracy the orbits of comets and other celestial bodies, understanding the nature of comets, and calculating the parallax of the sun. His calculations also contributed to the development of accurate longitude tables.[37]
In addition, Euler made important contributions in optics. He disagreed with Newton's corpuscular theory of light in the Opticks*, which was then the prevailing theory. His 1740s papers on optics helped ensure that the wave theory of light proposed by Christian Huygens would become the dominant mode of thought, at least until the development of the quantum theory of light.[38]*

Logic
He is also credited with using closed curves to illustrate syllogistic reasoning (1768). These diagrams have become known as Euler diagrams.[39]