

## Mathematics - Elective Pre-Calc. - Senior 1 – TEST 1 Nov.2 (60 min.) - p.1/3

**I – Linear Programming :** [30 pts] A publishing company is producing two kinds of software A and B on DVDs. It uses two machines for the production : one for the disk burning and one for the packaging.

Let  $x$  be the number of DVDs of A type and  $y$  be the number of DVDs of B type.

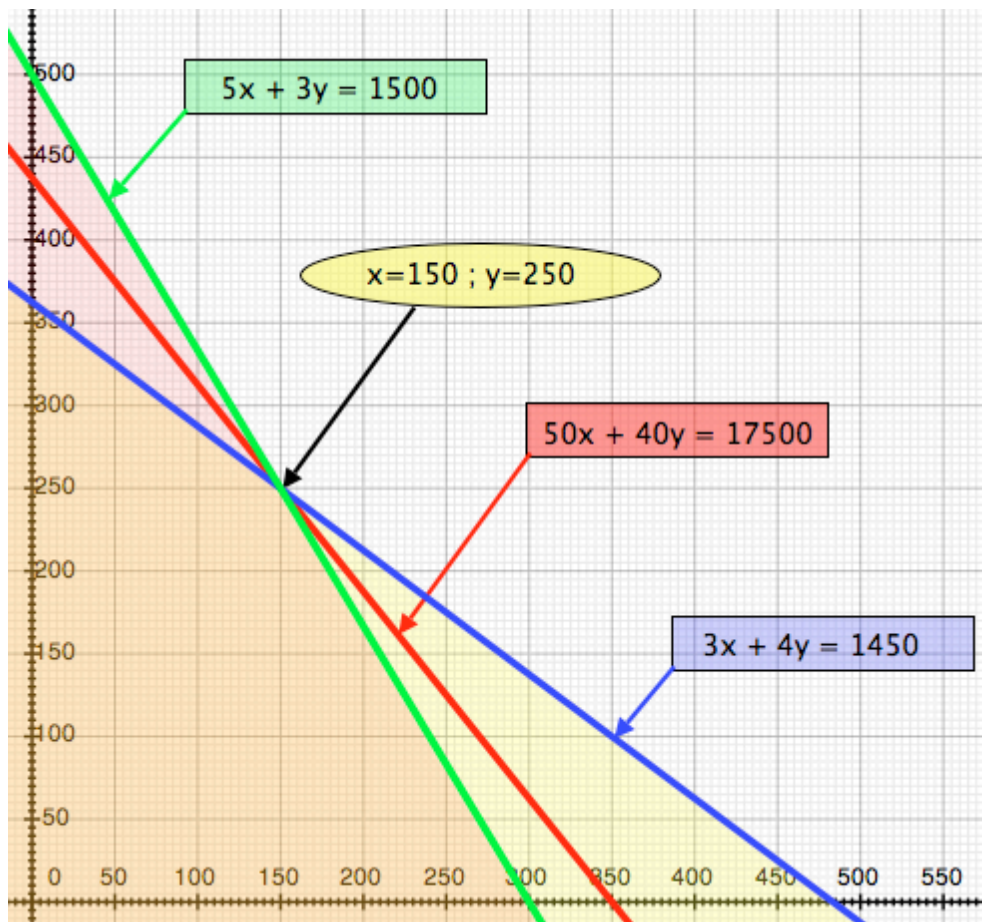
The burning machine takes 3 minutes to burn the DVD A and 4 minutes for B, but can work only for 24 hours and 10 minutes per series.

The packaging machine takes 5 minutes for the DVD A and 3 minutes for B, but it can work only for 25 hours in a row. Each DVD A is sold 50 Yuans and each DVD B is sold 40 Yuans.

1. Write the system of inequalities corresponding to this production.
2. Draw the lines corresponding to the production of each machine.
3. Shade the area corresponding to these conditions of production.
4. Write the equation corresponding to the total amount sold for this production.
5. Find the maximum number of DVD A and B which can be produced.
6. Draw the line of the sales corresponding to that maximum production.

- System of inequalities corresponding to the conditions of this production : 
$$\begin{cases} 3x + 4y \leq 1450 \\ 5x + 3y \leq 1500 \\ x \geq 0 ; y \geq 0 \end{cases}$$
- Total amount of money engaged for this production of  $x$  DVD/A and  $y$  DVD/B  

$$S = 50x + 40y$$
- $S$  is maximum for the point corresponding to the vertex of the authorized domain.  
 $x = 150$  DVD/A &  $y = 250$  DVD/B  $\Rightarrow S = 17500$  (Yuans) per series.



**II– Parabolas and Hyperbolas : [40 pts]**
 $f(x) = -\frac{1}{4}x^2 - x + 3$ 
 $g(x) = \frac{2x+12}{x+4}$ 

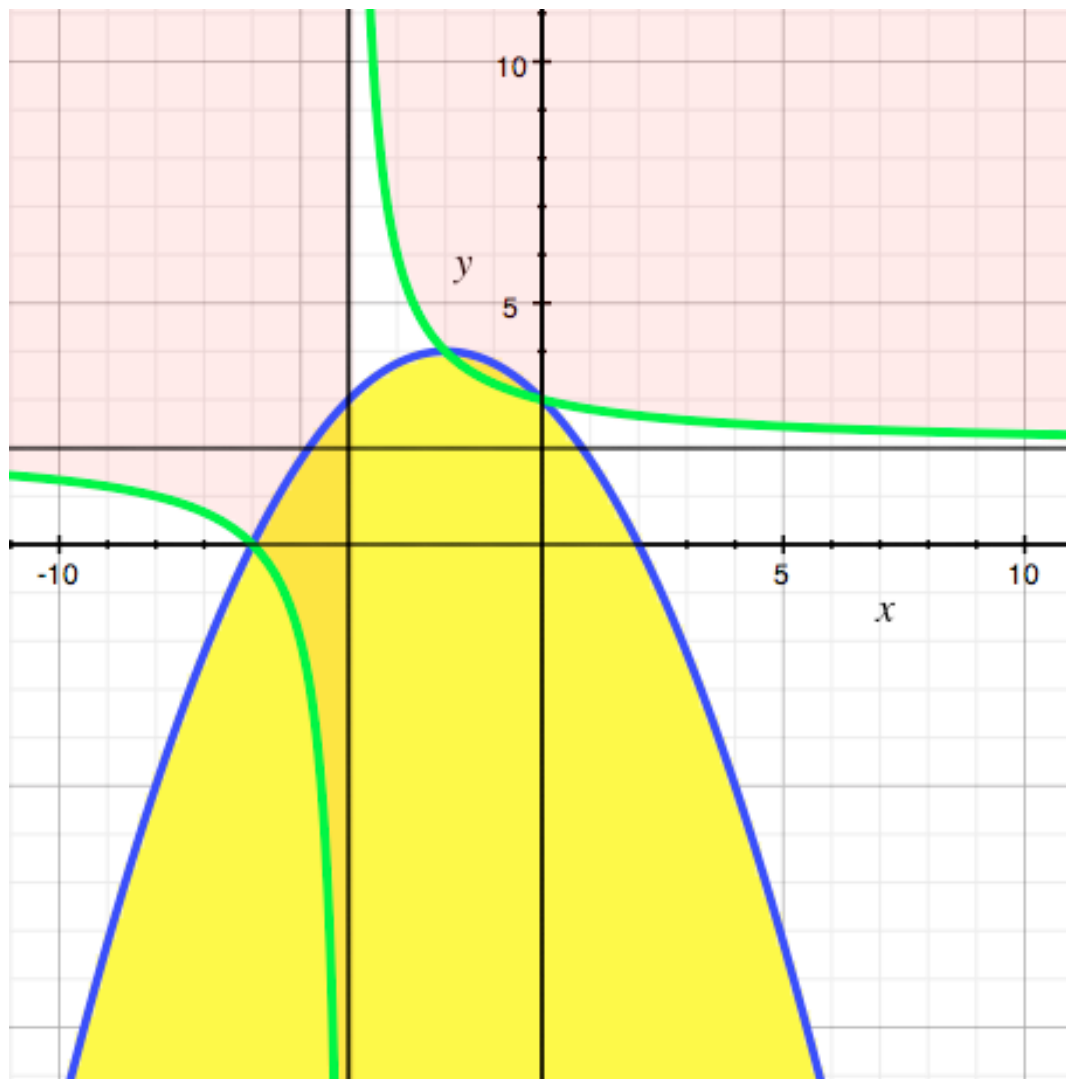
1. Draw carefully the graphs of the two functions in the same system of coordinates. Show the axis of symmetry of the Parabola and the asymptotes of the Hyperbola
2. Calculate and show the coordinates of intersections with the  $0x$  and the  $0y$  axes.
3. Solve the equation  $f(x) = g(x)$  to find the coordinates of the intersection points of the Parabola and the Hyperbola.
4. Shade the area of points  $(x;y)$  corresponding to the system of inequalities :  $y \leq f(x)$  &  $y \geq g(x)$

- Axis of Symmetry of the Parabola :  $x = -2$
- Center of Symmetry of the Hyperbola :  $(-4 ; 2)$
- Intersection of the Parabola with the  $x$  axis :  $y = 0 \Leftrightarrow x = -6$  or  $x = 2$
- Intersection of the Parabola with the  $y$  axis :  $x = 0 \Leftrightarrow y = 3$
- Intersection of the Hyperbola with the  $x$  axis :  $y = 0 \Leftrightarrow x = -6$
- Intersection of the Hyperbola with the  $y$  axis :  $x = 0 \Leftrightarrow y = 3$
- Intersection of the two curves :

$$-\frac{1}{4}x^2 - x + 3 = \frac{2x+12}{x+4} \Leftrightarrow -\frac{1}{4}(x+6)(x-2)(x+4) = 2(x+6)$$

$$\Leftrightarrow (x+6)\left[-\frac{1}{4}(x-2)(x+4) - 2\right] = 0 \Leftrightarrow (x+6)\left[-\frac{1}{4}x^2 - \frac{1}{2}x\right] = 0 \Leftrightarrow x(x+6)(x+2) = 0$$

$$\Leftrightarrow x = 0 (y = 3) \text{ or } x = -2 (y = 4) \text{ or } x = -6 (y = 0)$$



## III – Associated Functions and Transformations : graph the following functions in the system below

$$f_1(x) = \frac{1}{4}x^2 - |x| - 3 \quad ; \quad f_2(x) = -\sqrt{(x-3)^2 + 4} \quad ; \quad f_3(x) = -\sqrt{9+9x} + 6$$

- $f_1(x) = f(|x|)$  with  $f(x) = \frac{1}{4}x^2 - x - 3$ , because  $(|x|)^2 = x^2$ .  
Therefore the graph of  $f_1$  is associated to a parabola
- $f_2(x) = -\sqrt{(x-3)^2 + 4} = -|x-3| + 4$ . Therefore the graph of  $f_2$  is associated to the opposite of the absolute value function, translated from the origin by the vector  $V(3;4)$ .
- $f_3(x) = -\sqrt{9+9x} + 6 = -3\sqrt{x-(-1)} + 6$  Therefore the graph of  $f_3$  is associated to the function defined by the opposite of  $3\sqrt{x}$  and then translated from the origin by the vector  $V(-1;+6)$ .

