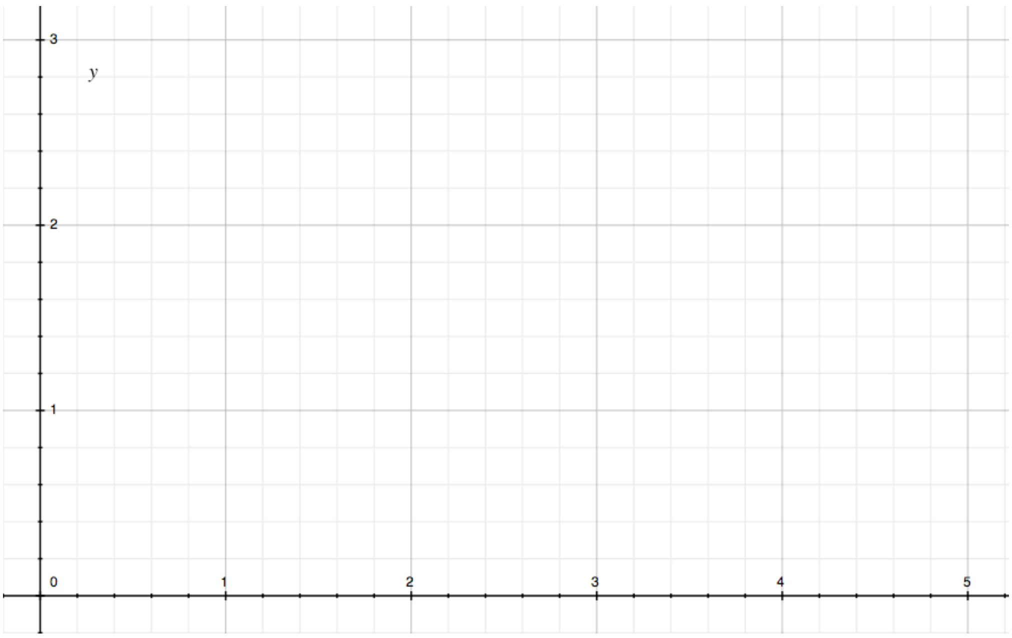
Numerical Sequences (2)

Problem I – Let $u_n = 2n + 3 - \sqrt{4n^2 + 4n + 5}$

1. *Calculate the first terms of (u_n).*
2. *Is (u_n) increasing or decreasing or neither ?*
3. *Is (u_n) bounded and if yes by which values ?*
4. *Prove that for any n>0 we have $|u_n - 2| \le \frac{1}{n}$.*
5. *What is the limit of (u_n) ?*

Problem I – Let $u_{n+1} = \sqrt{6 - u_n}$ defined by $u_{n+1} = f(u_n)$ *with* $f(x) = \sqrt{6 - x}$ *and* $u_0 = 5$

1. Graph the function f on [0 ; 6 [and draw the first terms of the sequence (u_n).
2. Find the coordinates of the intersection of (Cf) with the first bisector (y=x)
3. Indicate from the graph whether or not the sequence is :
   i. Monotonous (if yes how) :
   ii. Bounded (*if yes, what are the boundaries ?)*
   iii. Does-it seem to have a limit (*if yes which one is it?*)?



Problem III : Let f be the function defined by $f(x) = \dfrac{x+6}{x+2}$ for $x \geq 0$.

Study of the sequence (v_n) defined by $v_{n+1} = f(v_n) = \dfrac{v_n+6}{v_n+2}$; $n \geq 0$ and $v_0 = 5$.

4. Graph the function f on $[0 ; +\infty[$ and draw the first terms of the sequence (u_n).
Find the coordinates of the intersection of (Cf) with the first bisector (y=x)
Indicate from the graph whether or not the sequence is :
    i. Monotonous (if yes how) :
    ii. Bounded (*if yes, what are the boundaries ?)*
    iii. Does-it seem to have a limit (*if yes which one is it?*)?

5. Let $w_n = \dfrac{v_n - 2}{v_n + 3}$ *for any n>0.*

Show that the new sequence (w_n) is a geometric sequence :
    1. Find its first term and its reason.
    2. Find the expression of w_n directly in function of n.
    3. Deduct the limit of w_n.
    4. Find the expression of v_n in function *of* w_n
    5. Find the limit of v_n
    6. Check the *result on your graph.*