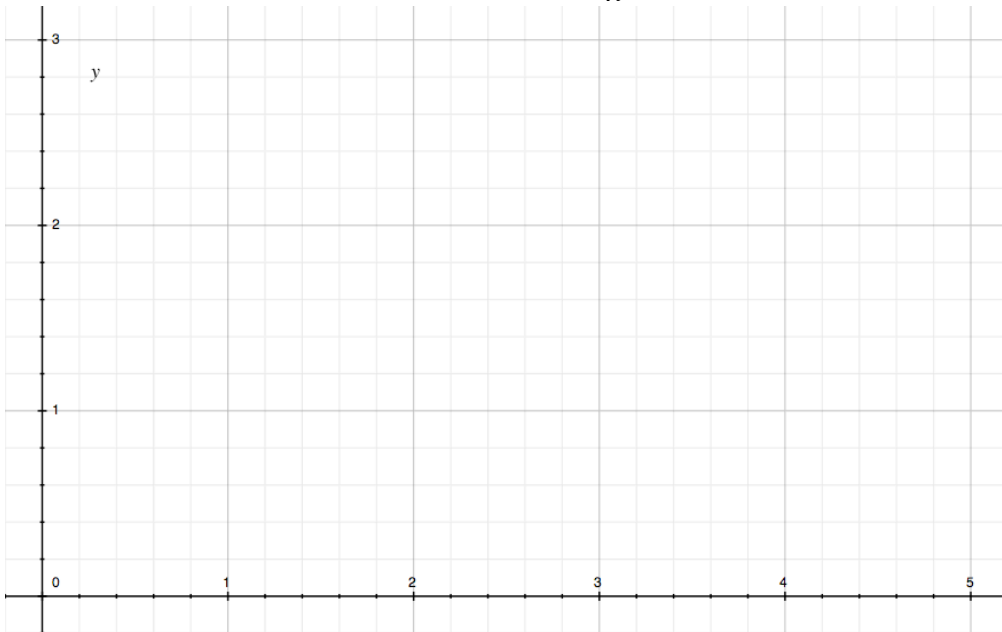


## Numerical Sequences

Problem I : Let  $f$  be the function defined by  $f(x) = \frac{2x+3}{x+4}$  for  $x \geq 0$ .

Study the Sequence defined by the formula  $u_n = f(n) = \frac{2n+3}{n+4}$  for every  $n \in \mathbb{N}$ .

- a. Graph the function  $f$  on  $[0 ; +\infty [$  and draw the first terms of the sequence  $(u_n)$ .  
Indicate from the graph whether or not the sequence is :
- Monotonous (if yes how) :
  - Bounded (if yes, what are the boundaries ?)
  - Does-it seem to have a limit (if yes which one is it?)?
- b. Prove that  $(u_n)$  is increasing
- c. Prove that  $(u_n)$  is bounded by 0 and 2.
- d. Find for which value of  $n$  we have :  $2 - \varepsilon < u_n < 2$  with  $\varepsilon = 10^{-2}$
- e. Prove that for any  $n \geq 1$  we have  $|u_n - 2| \leq \frac{5}{n}$ . Conclusion ?



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Problem II : Let  $f$  be the function defined by  $f(x) = \frac{2x+3}{x+4}$  for  $x \geq 0$ .

Study of the sequence  $(v_n)$  defined by  $v_{n+1} = f(v_n) = \frac{2v_n+3}{v_n+4}$  ;  $n \geq 1$  and  $v_0 = 4$ .

1. Graph the function  $f$  on  $[0 ; +\infty [$  and draw the first terms of the sequence  $(u_n)$ .  
 Find the coordinates of the intersection of  $(Cf)$  with the first bisector ( $y=x$ )  
 Indicate from the graph whether or not the sequence is :
  - i. Monotonous (if yes how) :
  - ii. Bounded (if yes, what are the boundaries ?)
  - iii. Does-it seam to have a limit (if yes which one is it)?

2. Let  $w_n = \frac{v_n - 1}{v_n + 3}$  for any  $n > 0$ .

Show that the new sequence  $(w_n)$  is a geometric sequence :

1. Find its first term and its reason.
2. Find the expression of  $w_n$  directly in function of  $n$ .
3. Deduct the limit of  $w_n$ .
4. Find the expression of  $v_n$  in function of  $w_n$
5. Find the limit of  $v_n$
6. Check the result on your graph.



3.