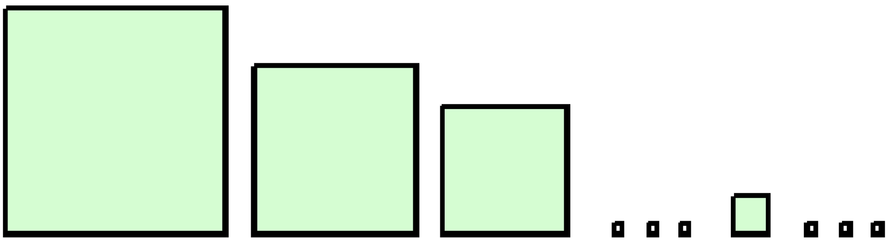
Numerical Sequences (3)

Problem I – The laughing cow

Suppose that the original box has a diameter of 10 cm, and that on each box the diameter of earrings box of the laughing cow is 1/5 of the previous one.

Find how many cows can be drawn until the diameter of the earrings box becomes less thsan1mm. *[let $d_0 = 10$ and d_n the diameter of the n^{th} box]*

Problem II – Square the squares



Suppose that the first square has a side of 10cm and that each square has a side of ¾ of the previous one. What is the area that can be covered by the total sum of the area of the infinite sequence of these squares.

Problem III Infinite decimals

Let a be a number (digit) between 1 and 9 included. Let $u_n = a.10^{-n}$, and $S_n = u_1 + u_2 + u_3 + \ldots + u_n$

1°) Write the expression of S_n as a function of n and a.

2°) Write the number N = 0.333 333 333 … 333 … in the previous form and prove that this number is a rational fraction to be found.

3°) Same problem with U = 0.999 999 999 … 999 …

Problem IV Recurrent sequences and reasoning by induction :

Let $u_{n+1} = \sqrt{6-u_n}$ defined by $u_{n+1} = f(u_n)$ *with* $f(x) = \sqrt{6-x}$ and $u_0 = 5$

We have studied the behavior of this sequence in Assignment #9 by constructing the first terms on the graph of the function *f* and the 1st bisector *(y=x)*. We have observed that the sequence (u_n) is not monotonous, is bounded by 0 and 5 and is converging towards 2.

Now we are going to prove it formally. (Not to be confused with the study of $u_n = f(n)$).

1°) show by induction that for any n, we have $0 \le u_n \le 5$,

2°) show that for any $n \ge 0$, $| u_n - 2 | \le ½ | u_{n-1} - 2 |$

3°) deduct from the previous inequality that for any $n \ge 0$, $| u_n - 2 | \le (½)^{n-1}$.

4°) Conclusion ?