Definition and Construction of an Equilateral Hyperbola (Part 1)

Let f be the function defined by : $f : x \mapsto y = \dfrac{A}{x}$

I- **Algebraic properties**:

1°) **Odd** function: for any x ∈ ℝ*, f(-x) = - f(x).

2°) Rate of growth ***non constant*** : $T_{[f,(x_1,x_2)]} = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{-A}{x_1 x_2}$

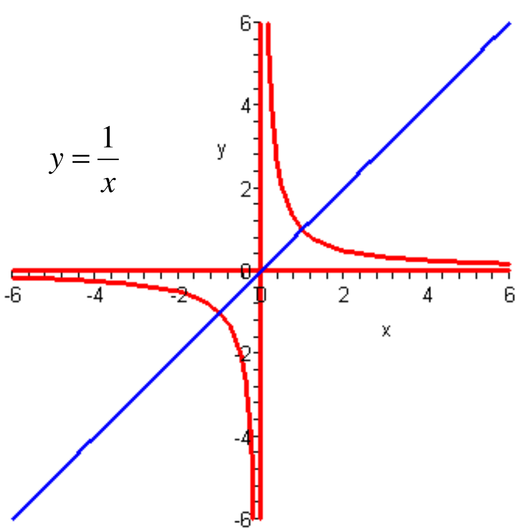
3°) Sign of T = Signe of *(- A)* on [0 ; +∞[and on]- ∞ ; 0]

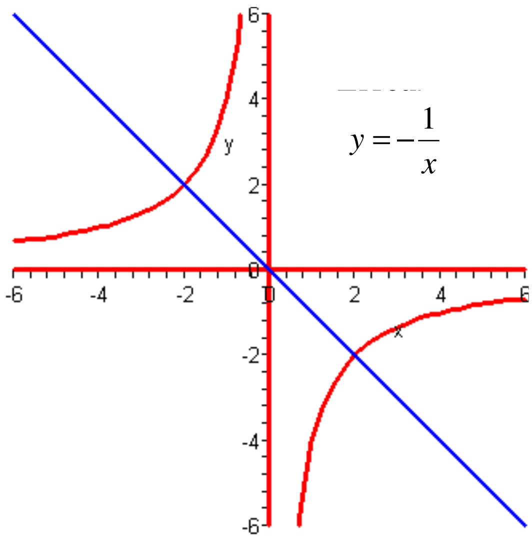
4°) Variation Chart :

	A > 0			***A < 0***	
x	-∞ -1 0	0 1 +∞	*x*	-∞ -1 0	0 1 +∞
T	-	-	T	+	+
f	$0^{(-)}$ ↘ -A ↘ - ∞	+ ∞ ↘ A ↘ $0^{(+)}$	*f*	$0^{(+)}$ ↗ -A ↗ +∞	- ∞ ↗ A ↗ $0^{(-)}$

II- **Geometric Properties** :

1°) The curve representing f is symmetrical through the Origin of axes. This curve is called an **Equilateral Hyperbola** because of its central Symmetry and because x and y vary in reverse directions. (The word hyperbolic means something exaggerated)
2°) The Hyperbola cuts the 1st bisector ($y = x$) in I $(\sqrt{A};\sqrt{A})$ *if* $A > 0$ or $(\sqrt{-A};-\sqrt{-A})$ *if* $A < 0$
3°) On I the Hyperbola is tangent to the line perpendicular to the bisector.
4°) The Hyperbola contains the point J(1 ;A) and its symmetrical point (-1 ;-A) through O
5°) If A > 0 the 1st bisector ($y = x$) is an axis of symmetry.
If A < the 2nd bisector ($y = - x$) is an axis of symmetry.
6°) When |A| is very large compared to 1 (|A| >> 1), The Hyperbola is very wide and away from O. Inversely if |A| <<1 the Hyperbola is very narrow and close to 0.
7°) The Hyperbola contains absolutely no segment of a straight line.
8°) The Hyperbola has two *asymptotes* which are the axes of coordinates (Ox) et (Oy).





Hyperbolas & Homographic Functions (Part 2)

Homographic functions are those defined by the type : $f : x \mapsto y = \dfrac{ax+b}{cx+d}$ *with* $\boldsymbol{c \neq 0}$

That expression can take one or the other of the following forms :

$$(H_1) \quad y = \frac{A}{x}$$

$$(H_2) \quad y = \frac{A}{x} + H$$

$$(H_3) \quad y = \frac{A}{x-L}$$

$$(H_4) \quad y = \frac{A}{x-L} + H$$

$$(H_5) \quad y = \frac{ax+b}{cx+d}$$

1°) Transformation from (H_1) to (H_2) is a **Translation** defined by the vertical vector $H\vec{j}$ (parallel to the (O*y*) axis. (H_2) intercepts (O*y*) in y = H. *(H = « Hight » ; L = « Length »)*

2°) Transformation from (H_1) to (H_3) is a **Translation** defined by the horizontal vector $L.\vec{i}$ (parallel to the (O*x*) axis).

3°) Transformation from (H_1) to (H_4) is a **Translation** of vector $\vec{V} = L.\vec{i} + H.\vec{j}$

4°) The Hyperbola (H_4) has its center of symmetry in O'(L ;H).
Let **X = *x* – L** and **Y = *y* – H** then **Y = A/X** which means that (H_4) is Symmetrical through O'
Therefore (H_4) is drawn in the system (O'X ; O'Y) just like (H_1) in the system (*Ox,Oy*).

5°) To build the Hyperbola (H_5) one can choose between two methods :

a. Change (H_5) into (H_4) by breaking the fractions in simple elements (cf. examples).

b. Find the coordinates of the center with the formulas $O'(\,L = \dfrac{-d}{c}\,;H = \dfrac{a}{c}\,)$

then find the intersections with the two axes : (Oy) : $(0\,;\frac{b}{d})$ and (Ox) $(\frac{-b}{a}\,;0)$.

$$y = \frac{x-2}{x+2} = \frac{-4}{x+2} + 1$$