jiguanglaoshi@gmail.com - http://beijingshanmaths.org

Extended integrals

I. $\int_a^b f(t)\,dt = \lim\limits_{x\to b} \int_a^x f(t)\,dt$ or $\int_a^{+\infty} f(t)\,dt = \lim\limits_{x\to +\infty} \int_a^x f(t)\,dt$

Example : $F(x) = \int_2^x \ln\frac{t-1}{t+1}\,dt$ and $\lim\limits_{x\to 1^+} F(x) = \ln\frac{27}{4}$

then although the fonction under the Sum Sign is not defined for t=1,

we may write $\int_2^1 \ln\frac{t-1}{t+1}\,dt = \ln\frac{27}{4}$

Similaly we have $\lim\limits_{x\to +\infty} F(x) = -\infty$ then : $\lim\limits_{x\to +\infty} \int_2^x \ln\frac{t-1}{t+1}\,dt = \int_2^{+\infty} \ln\frac{t-1}{t+1}\,dt = -\infty$

II. **Exercise** : let $f(t) = (t-1)e^{-t} + 1$ and $F(x) = \int_0^x f(t)\,dt$

1. Study of the function *f* :
   i. Limits
   ii. Derivative
   iii. Variations
   iv. Graph
2. Study of the function F
   i. give *F'(x)*
   ii. Use the IBP formula to calculate *F(x)* for any *x* ∈]-∞ ; +∞[.
   iii. Find $\lim\limits_{x\to +\infty} F(x) = \int_0^{+\infty} f(t)\,dt$
   iv. Find $\lim\limits_{x\to -\infty} F(x) = \int_0^{-\infty} f(t)\,dt$

III. **Let** $G(x) = \int_0^{u(x)} f(t)\,dt$ and u(x) be a derivable function.

i. Calculate *G'(x) in general form*
ii. *Let u(x) =* x^2*, give G'(x).*