

Study of the function f defined by $f(0) = 1$ and for $x \neq 0$ by $f(x) = \left(1 + \frac{4}{x^2}\right)^{\frac{x}{2}}$

I - 1. Complete the formula : $\lim_{x \rightarrow \pm\infty} x \ln \left(1 + \frac{1}{x}\right) = \dots$ and write the **complete proof** below 3pts

2. Use the previous result to find $\lim_{x \rightarrow \pm\infty} x \ln \left(1 + \frac{1}{x^2}\right) = \dots$ with **complete proof** below 2pts

3. Complete the formula : $\lim_{x \rightarrow 0^+} x \ln x = \dots$ with **complete proof** below : 2pts

4. Use the previous result to find $\lim_{x \rightarrow 0} x \ln \left(1 + \frac{1}{x^2}\right) = \dots$ with **complete proof** below : 3pts

II - Let, for $x \neq 0$, $U(x) = x \ln \left(1 + \frac{4}{x^2}\right)$.

Calculate the derivative ($x \neq 0$), $U'(x) =$

Let $V(x) = U'(x)$, ($x \neq 0$) **calculate** $V'(x)$: 2pts

Chart the **sign** of $V'(x)$ and the **variations** of V on \mathbb{R}^* with the **limits** and the **Minima** values. Explain why $V(x)$ has two **zeroes** a and b and place them. 6pts

x	$-\infty$	$+\infty$
<i>Sign</i> [$V'(x)$]		
<i>Variations and limits of</i> $V(x)$		
<i>Sign of</i> $U'(x)$		

III - Study the **variations of f** :1. Show that the sign of $f'(x)$ is the same as that of $U'(x)$

1 pt

2. Study the **limits of U(x) and f(x)** : [indicate which formula is used]a. Show the limit of $U(x)$ and give the limit of $f(x)$ at $x = 0$

2pts

b. Show the limits of $U(x)$ and give the limit of $f(x)$ in $+\infty$ and $-\infty$

2pts

3. Complete the **chart** of f with all the previous results :[one can admit that $a \approx -1$; $b \approx 1$; $f(-1) \approx 0.4$; $f(1) \approx 2.2$]

2pts

x	$-\infty$	$+\infty$
Sign [$f'(x)$]		
Variations and limits of $f(x)$		

4. **Graph** of the function f (show the asymptotes if any)

3pts

