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Problem I :

$$f(x)=\frac{x^3+2x^2}{2(x-1)^2}$$

1. Break *f(x)* into the form $ax+b+\frac{cx+d}{2(x-1)^2}$ *(x≠1)* .... 4 pts

By Euclidian division or by identification we get $f(x)=\frac{1}{2}x+2+\frac{7x-4}{2(x-1)^2}$ *for x ≠ 1*

2. Find the limits of *f* at the ends of each interval of its definition set .... 4 pts

$\lim_{x\to+\infty} f(x)=\lim_{x\to+\infty}\frac{x^3}{2x^2}=\lim_{x\to+\infty}\frac{x}{2}=+\infty$ *and similarly* $\lim_{x\to-\infty} f(x)=\lim_{x\to-\infty}\frac{x}{2}=-\infty$

and $\lim_{x\to1^\pm} f(x)=\lim_{x\to1^\pm}\frac{3}{2(x-1)^2}=+\infty$ *hence x = 1 is a"vertical " asymptote.*

3. Show that *(C$_f$)* has a "vertical" and an oblique asymptote (Δ), give their equations. .... 4 pts

from (1) we get $\lim_{x\to+\infty}[f(x)-(\frac{1}{2}x+2)]=\lim_{x\to+\infty}\frac{7x-4}{2(x-1)^2}=\lim_{x\to+\infty}\frac{7x}{2x^2}=\lim_{x\to+\infty}\frac{7}{2x}=0^+$

and similarly $\lim_{x\to-\infty}[f(x)-(\frac{1}{2}x+2)]=\lim_{x\to-\infty}\frac{7}{2x}=0^-$

4. Justify the position of *(C$_f$)* with respect to this asymptote. .... 2 pts

From the sign of the above limits we can say that (C$_f$) is above (Δ) in +∞ and under (Δ) in -∞

5. Find the derivative *f '(x)* and factor it in binomials .... 6 pts

$$f'(x)=\frac{(3x^2+4x)2(x-1)^2-(x^3+2x^2)4(x-1)}{4(x-1)^4}=\frac{x^3-3x^2-4x}{2(x-1)^3}=\frac{x(x+1)(x-4)}{2(x-1)^3}$$ *for x ≠ 1*

6. Give the zeroes of *f '(x)* and justify the signs of the derivative. .... 6 pts

f '(x) = 0 ⇔ x= 0 or x = -1 or x = 4 and Sign[f '(x)] = Sign x(x-1)(x+1)(x-4) for x ≠ 1

7. Find the intersections of *(C$_f$)* with the axes of coordinates. .... 4 pts

f(x) = 0 ⇔ x= 0 or x = -2 and f(0) = 0

8. Find the intersection of *(C$_f$)* with its asymptote (Δ). .... 2 pts

$f(x)=\frac{1}{2}x+2 \Leftrightarrow \frac{7x-4}{2(x-1)^2}=0 \Leftrightarrow x=\frac{4}{7}$ and then y $=\frac{1}{2}\frac{4}{7}+2=\frac{16}{7}\simeq 2.3$ Point I(0.4 ; 2.3).

9. Find the equation of the tangent line in *A(-2;0)* to *(C$_f$)* .... 2 pts

f '(-2) = 2/9 there for the equation of (T$_A$) is : y = 2/9 (x + 2) = 2/9 x + 4/9

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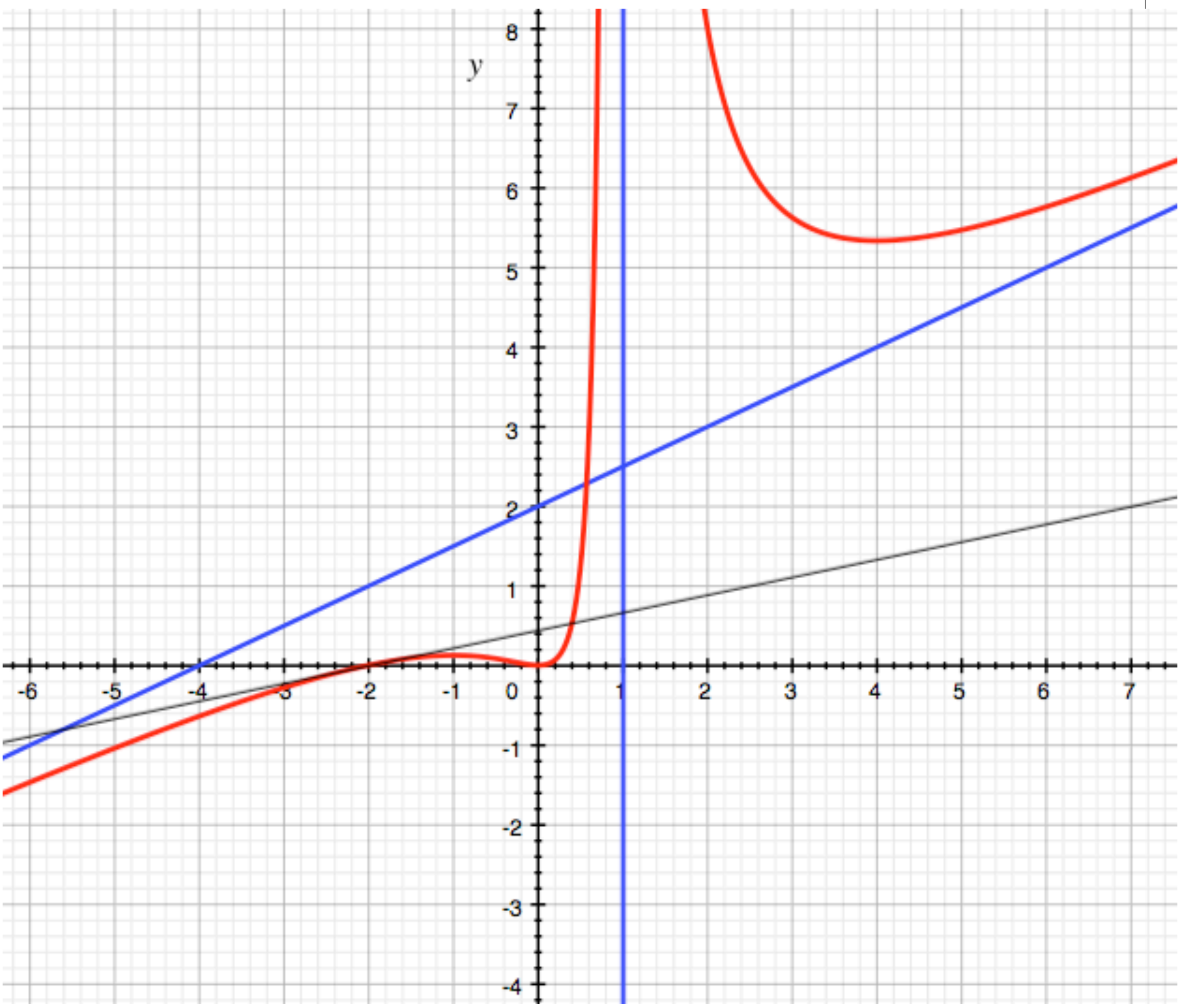
10. Show <u>all</u> the previous results in the chart below. .... 6 pts

x	$-\infty$		-2		-1		0		1		4		$+\infty$
Sign [f '(x)]		+	0	+	0	_	0	+	||	_	0	+	
Variations *and* ***limits*** of *f*	$-\infty$	↗	0	↗	M	↘	m_1	↗	$+\infty$||$+\infty$	↘	m_2	↗	$+\infty$

11. Give the approximate decimal value of the extremes here : .... 2 pts

$$M = f(-1) = 1/8 = 0.125 \;;\; m_1 = f(0) = 0, \quad \text{and } m_2 = f(4) = 16/3 = 5.3$$

12. Draw carefully (C_f) its asymptotes, and the tangent line (T_A). .... 8 pts



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Problem I : $$f(x)=\frac{2x^2-x^3}{2(x+1)^2}$$

13. Break *f(x)* into the form $ax+b+\frac{cx+d}{2(x+1)^2}$ *(x≠1)* .... 4 pts

By Euclidian division or by identification we get $f(x)=-\frac{1}{2}x+2-\frac{7x+4}{2(x+1)^2}$ *for x ≠ -1*

14. .... F
ind the limits of *f* at the ends of each interval of its definition set .... 4 pts

$\lim_{x\to+\infty} f(x)=\lim_{x\to+\infty}\frac{-x^3}{2x^2}=\lim_{x\to+\infty}\frac{-x}{2}-\infty$ and similarly $\lim_{x\to-\infty} f(x)=\lim_{x\to-\infty}\frac{-x^3}{2x^2}==\lim_{x\to-\infty}\frac{-x}{2}=+\infty$

and $\lim_{x\to-1^\pm} f(x)=\lim_{x\to-1^\pm}\frac{3}{2(x+1)^2}=+\infty$ *hence x =-1 is a"vertical " asymptote.*

15. Show that *(C_f)* has a "vertical" and an oblique asymptote (Δ), give their equations. .... 4 pts

from (1) we get $\lim_{x\to+\infty}[f(x)-(-\frac{1}{2}x+2)]=\lim_{x\to+\infty}\frac{-7x-4}{2(x+1)^2}=\lim_{x\to+\infty}\frac{-7x}{2x^2}=\lim_{x\to+\infty}\frac{-7}{2x}=0^-$

and similarly $\lim_{x\to-\infty}[f(x)-(-\frac{1}{2}x+2)]=\lim_{x\to-\infty}\frac{-7}{2x}=0^+$

16. Justify the position of *(C_f)* with respect to this asymptote. .... 2 pts

From the sign of the above limits we can say that (C_f) is above (Δ) in -∞ and under (Δ) in +∞

17. Find the derivative *f '(x)* and factor it in binomials .... 6 pts

$$f'(x)=\frac{(4x-3x^2)2(x+1)^2-(2x^2-x^3)4(x+1)}{4(x+1)^4}=\frac{-x^3-3x^2+4x}{2(x+1)^3}=-\frac{x(x-1)(x+4)}{2(x+1)^3}\ \textit{for } x\neq -1$$

18. Give the zeroes of *f '(x)* and justify the signs of the derivative. .... 6 pts

f '(x) = 0 ⇔ x= 0 or x = 11 or x = -4 and Sign[f '(x)] = Sign x(x+1)(x-1)(x-4) for x ≠ -1

19. Find the intersections of *(C_f)* with the axes of coordinates. .... 4 pts

f(x) = 0 ⇔ x= 0 or x = 2 and f(0) = 0

20. Find the intersection of *(C_f)* with its asymptote (Δ). .... 2 pts

$f(x)=-\frac{1}{2}x+2 \Leftrightarrow \frac{7x+4}{2(x+1)^2}=0 \Leftrightarrow x=-\frac{4}{7}$ and then y $=\frac{1}{2}\frac{4}{7}+2=\frac{16}{7}\simeq 2.3$ Point I(-0.4 ; 2.3).

21. Find the equation of the tangent line in *A(2;0)* to *(C_f)* .... 2 pts

f '(2) = -2/9 there for the equation of (T_A) is : y =-2/9 (x + 2) =-2/9 x + 4/9

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22. Show <u>all</u> the previous results in the chart below. .... 6 pts

x	$-\infty$		-4		-1		0		1		$+\infty$
Sign [f '(x)]		_	0	+	||	_	0	+	||	_	
Variations *and **limits*** of *f*	$+\infty$	↘	M	↗	$+\infty$||$+\infty$	↘	m_1	↗	m_2	↘	$-\infty$

23. Give the approximate decimal value of the extremes here : .... 2 pts

$$M = f(-4) = 16/3 = 5.3 \; ; \; m_1 = f(0) = 0 \; ; \text{ and } m_2 = f(1) = 1/8 = 0.125$$

24. Draw carefully (C_f) its asymptotes, and the tangent line (T_A). .... 8 pts