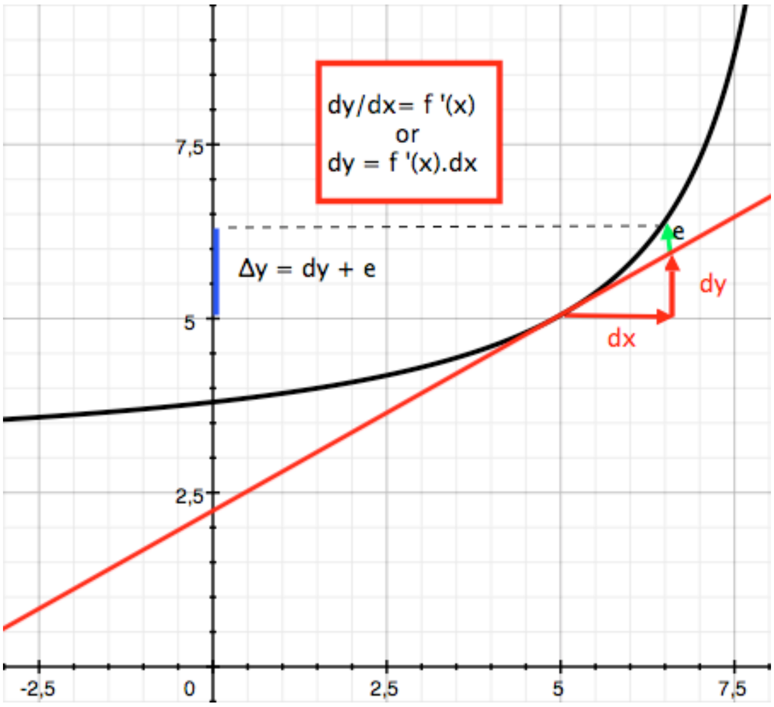
Derivative vs Differential



Definition : *the differential* ***dy*** *of a numerical function in one variable on a given point x is equal to the variation of the ordinate measured* ***on the tangent line*** *at that point, for the same small variation* ***dx*** *of the variable.*

We may define the ***differential*** *as the "****principal part of the variation*** **Δy***"when dx is "small enough".*

$$f'(x) = \lim_{dx\to 0} \frac{f(x+dx)-f(x)}{dx} \quad \therefore \quad \frac{f(x+dx)-f(x)}{dx} = f'(x)+e$$

$$(e \to 0,\ when\ dx \to 0)$$

$$\therefore \quad f(x+dx)-f(x) = f'(x).dx + e.dx,$$

$$Let\ \Delta y = f(x+dx)-f(x)\ , \quad then\ for\ a\ \text{small}\ value\ of\ dx:\ \Delta y \simeq f'(x).dx$$

The number $f'(x).dx$ *is noted* dy *and is called the* differential *of* f *at* x.

$$therefore: \quad dy = f'(x).dx \quad \text{or} \quad \frac{dy}{dx} = f'(x)$$

Example : *let* $f(x) = \dfrac{2.8x - 34.2}{x-9}$ and $f'(x) = \dfrac{9}{(x-9)^2}$; $x = 5, f(5) = 5.05,\ f'(5) = \dfrac{9}{16} = 0.506 \approx 0.51$

then to find an approximate value of f(5.5) we may use the differential of f at x=5, with dx = +0.5 ,

then dy = f '(5).dx = 0.51 x 0.5 ≈ 0.25, therefore Δy ≈ 0.25, then f(5.5) – f(5) ≈ 0.25,

finally f(5.5) ≈ f(5) + 0.29 ≈ 5.05 + 0.25 = 5.30.

With direct calculation we would find with a calculator f(5.5) = (2.8 x 5.5 – 34.2) / (5.5 – 9) = 5.37

The error is less than 0.1, which is considered as very satisfactory, with respect of the picture.