jiguanglaoshi@gmail.com - http://beijingshanmaths.org

∫ Extra change of variables ∫

I. Use an adequate change of variable to calculate the following integrals :

1. $I = \int_0^1 \sqrt{1-x^2}\,dx$

2. $J = \int_0^1 \frac{dx}{\sqrt{1-x^2}}$

3. $K = \int_0^1 \frac{dx}{1+x^2}$

II. Without calculating the value of each integral, prove by an adequate change of variable that :

1. $I = \int_0^{\frac{\pi}{2}} (\sin x)^3 \cos x\,dx$ and $J = \int_0^{\frac{\pi}{2}} (\cos x)^3 \sin x\,dx$, prove that $I = J$

2. $K_n = \int_{-1}^1 (1-x^2)^n\,dx$ and $K'_n = \int_{-1}^1 (x^2-1)^n\,dx$, prove that $K'_n = (-1)^n K_n$

3. $I_n = \int_0^{\frac{\pi}{2}} (\sin x)^n \, dx$ and $J_n = \int_0^{\frac{\pi}{2}} (\cos x)^n \, dx$, prove that $I_n = J_n$

4. $K_n = \int_{-1}^{1} (1-x^2)^n \, dx$ and $I_n = \int_0^{\frac{\pi}{2}} (\sin x)^n \, dx$, prove that $K_n = 2\, I_{2n+1}$

5. Calculate $F(x) = \int_x^1 \ln\left(1+\frac{1}{t}\right) dt$ and $G(x) = \int_{-1}^{x} \ln\left(1-\frac{1}{t}\right) dt$, prove that $G(x) = F(-x)$

6. Calculate $F(x) = \int_0^x t^2 e^t \, dt$ and $G(x) = \int_0^x t^2 e^{-t} \, dt$, prove that $G(x) = -F(-x)$

III. Just for fun …

1. Prove that $I = \int_0^{+\infty} t e^{-pt} \, dt = \frac{1}{p^2}$

2. Prove that $I_{(p,n)} = \int_0^{+\infty} t^n e^{-pt} \, dt = \frac{n!}{p^{n+1}}$