

Finite Variations Inequalities

and their applications on sequences

Let f be the function defined by : $f(x) = 1 + \frac{x-1}{\sqrt{x^2+1}}$ and $u_{n+1} = f(u_n)$ with $u_0 = 2$.

- 1° Study the limits of $f(x)$ on $] -\infty ; +\infty [$ and give the equations of the asymptotes.
 - 2° Calculate $f'(x)$ and study it's sign and give the variations of f .
 - 3° Show that there is at least on fixed point for f , such that $f(x) = x$.
 - 4° Draw the graph of f on $[-2 ; 2]$ and show the construction of the first terms of (u_n)
 - 5° Research of a majorant M , $0 < M < 1$ for $|f'(x)|$ on $[1;2]$:
 - i. Calculate $f''(x)$, second derivative of f , on $[1;2]$
 - ii. Study the Sign of $f''(x)$ and chart the variations of $f'(x)$ on $[1;2]$.
 - iii. Show that for $x \in [1;2]$, $|f(x)| \leq 1/\sqrt{2}$
 - 6° Use the Finite Variations Inequalities to prove that $\lim u_n = 1$.
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