jiguanglaoshi@gmail.com 

Jingshan School of Beijing 

# Definition & Construction of a Parabola (Part 1)

Let *f* be the function defined by : $f : x \mapsto ax^2$ (a≠0)

## I- **Algebraic properties** :

1°) ***Even** function* : for any $x \in \mathbb{R}$, $f(-x) = f(x)$.

2°) Rate of growth ***non constant*** : $T_{[f,(x_1;x_2)]} = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = a(x_2 + x_1)$

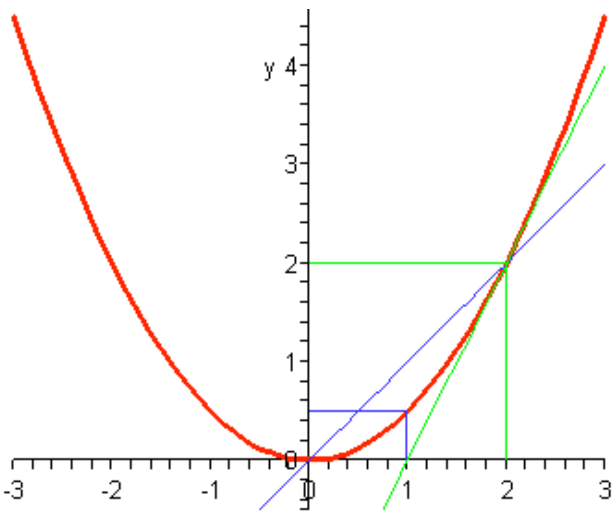
3°) Sign of T = Sign of *a* on [0 ; +∞[ , and Sign of T = Sign of (-*a*) on ]- ∞ ; 0]

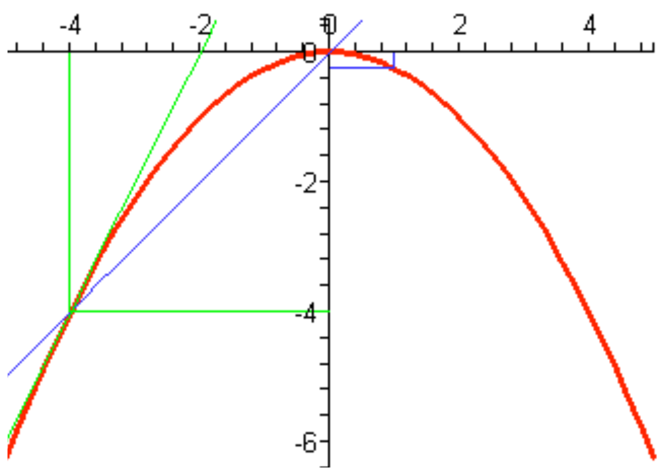
4°) Chart of the Variations of *f* :

| $a > 0$ | | | | | | $a < 0$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| x | -∞ | -1 | 0 | 1 | +∞ | x | -∞ | -1 | 0 | 1 | +∞ |
| T | | - | ‖ | + | | T | | + | ‖ | - | |
| f | +∞ | ↘ a | 0 | a ↗ | +∞ | f | - ∞ | ↗ a | 0 | a ↘ | - ∞ |

## II- **Geometric Properties** :

1°) The curve has (Oy) as an axis of symmetry. For that reason the curve is called a **Parabola.**
2°) The Parabola is tangent to the (Ox) Axis in O.
3°) The Parabola passes through the point A(1 ; a).
4°) If a > 0 the Parabola concavity is directed towars the positive y :
*(*as one can say the *« the bowl can hold water »)*
If a < 0 the Parabola concavity is directed towards the négative y :
*(*as one can say the *« the bowl cannot hold water »)*
5°) The Parabola intercepts the 1st bisector line (y = x) at point B(1/a ; 1/*a*)
6°) On B the Parabola is tangent to the line joining B to the middle of the segment located under the tangent, which is the point of abscissa 1/2*a*
7°) By symmetry with respect to Oy we get the points A'(-1 ;*a*) and B'(-1/*a* ; 1/*a*)
8°) If a is very small compared to the unity (*a* << 1) , the parabola is widely opened, inversely if *a* >>1 the parabola is very narrow around the axis of symmetry.
9°) The parabola contains absolutely no piece of a straight line.
10°) The branches spread indefinitely in the direction of the Oy axis.



# Second degree functions (Part 2)

Second degree functions are in the general form : $f : x \mapsto ax^2 + bx + c$ *with* $a \neq 0$
This expression can take any of the following forms :

| | |
|---|---|
| **($P_1$)** | $y = a\,x^2$ |
| **($P_2$)** | $y = a\,x^2 + H$ |
| **($P_3$)** | $y = a\,(x - L)^2$ |
| **($P_4$)** | $y = a\,(x - L)^2 + H$ |
| **($P_5$)** | $y = a\,(x - x')(x - x'')$ |
| **($P_6$)** | $y = a\,x^2 + bx + c$ *(trinomial)* |

1°) Transformation from ($P_1$) to ($P_2$) is a **Translation** defined by the vertical vector $H\vec{j}$ (parallel to the (Oy) axis. ($P_2$) intercepts (Oy) in y = H. *(H = « Hight » ; L = « Length »)*
2°) Transformation from ($P_1$) to ($P_3$) is a **Translation** defined by the horizontal vector $L.\vec{i}$ (parallel to the (O*x*) axis)
3°) Transformation from ($P_1$) to ($P_4$) is a **Translation** of vector $\vec{V} = L.\vec{i} + H.\vec{j}$
The Parabola ($P_4$) has a vertex in O'(L ;H).
Let **X = *x* – L** and **Y = *y* – H** then **Y = a X²** which means that ($P_4$) is Symmetrical whith respect of the axis defined by x = L (parallel to (Oy))
($P_4$) is drawn in the system (O'X,O'Y) just like ($P_1$) in the system (*Ox*,*Oy*).
4°) The Parabola ($P_5$) intercepts the axis (Ox) in x' and x'', its vertex is then at
S of abscissa = $\frac{x' + x''}{2} = \frac{b}{2a}$ ordinate H = *f*(L).
5°) To build the parabola ($P_6$) one can either :
a. use the form ($P_4$) by breaking the trinomial in that « canonic » form.
b. find the coordinates of the vertex **O'{L=- b/2a ; H=*f*(L) }** then find the Ox and Oy intersection pts **:** on (Oy) : (x = 0 ; y = c) and (Ox) solutions of the équation $a\,x^2 + bx + c = 0$ *(if any)*.

***Example*** *: let (P) be the Parabola defined by* $y = 1/4\,(x - 2)^2 + 3$ then L=2 ; H=3 ; Y = ¼ X²