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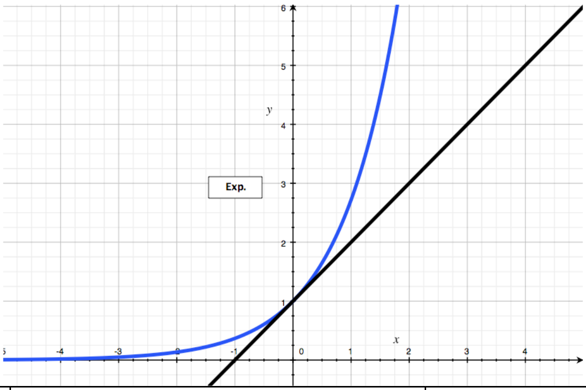
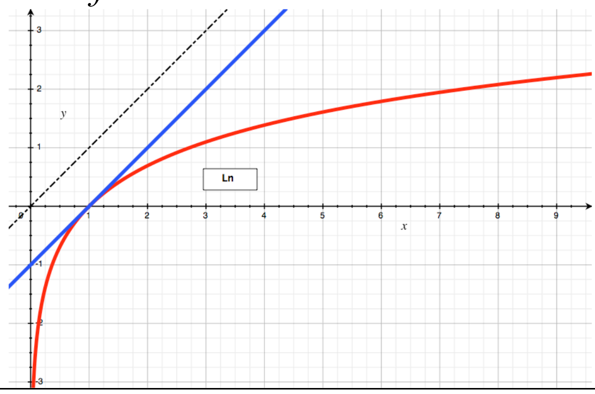
Exponential – Power – Logarithm

Limits and comparison of limits





	Limits	Demo
(1)	$\lim\limits_{x\to+\infty} Exp(x) = +\infty$	*Let U(x) = Exp(x) – (x+1) ; U'(x) = Exp(x) – 1 =* $e^x - 1$ $\therefore$ *U'(x) >0 on [0 ; +∞[and U'(x) <0 on]-∞;0]* $\therefore$ *U is decreasing on]-∞;0] and increasing on [0 ; +∞* *and U(0) = 0 is a Minimum of U(x) on]-∞ ; +∞[* $\therefore U(x) \geq 0$ *on]-∞ ; +∞[* $\therefore e^x \geq x+1 > x$ *then (1).* *NB : The curve is always above the tangent @(0;1).*
(2)	$\lim\limits_{x\to-\infty} Exp(x) = 0^+$	$e^x = \dfrac{1}{e^{(-x)}}$; $(-x)\to+\infty \therefore e^{(-x)} \to +\infty \therefore \dfrac{1}{e^{(-x)}} \to 0^+$
(3)	$\lim\limits_{x\to0} Exp(x) = 1$	*The Exponential function being defined as the only function that's equal to it's derivative, is continuous on R,* $\therefore \lim\limits_{x\to0} Exp(x) = Exp(0) = 1$
(4)	$\lim\limits_{x\to0} \dfrac{e^x - 1}{x} = 1$	$\lim\limits_{x\to0} \dfrac{e^x-1}{x} = \lim\limits_{x\to0} \dfrac{Exp(x)-Exp(0)}{x-0} = Exp'(0) = Exp(0) = 1$
(5)	$\lim\limits_{x\to+\infty} \ln x = +\infty$	$\forall M > 0, \exists A > 0 \;\; A = e^M \Leftrightarrow M = \ln(A)$ $x > A \Rightarrow \ln x > \ln(A) \Leftrightarrow \ln x > M$ *By definition this means (5)*
(6)	$\lim\limits_{x\to0^+} \ln(x) = -\infty$	$\ln(x) = -\ln\dfrac{1}{x}$; $\lim\limits_{x\to0^+} \dfrac{1}{x} = +\infty \Rightarrow \lim\limits_{x\to0^+} \ln\dfrac{1}{x} = +\infty \Rightarrow (6)$
(7)	$\lim\limits_{x\to-\infty} \ln(x) =$ None !	*The function ln is not defined for x* ≤ 0
(8)	$\lim\limits_{x\to0} \dfrac{\ln(1+x)}{x} = 1$	$\lim\limits_{x\to0} \dfrac{\ln(1+x)}{x} = \lim\limits_{x\to0} \dfrac{\ln(1+x)-\ln(1)}{x} = \ln'(1) = \dfrac{1}{1}$

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II - Comparisons : $(n \geq 2)$

	Limits	Demo
(9)	$\lim\limits_{x\to+\infty}\frac{Exp(x)}{x}=+\infty$	$e^{\frac{x}{2}}>\frac{x}{2}\Rightarrow\left(e^{\frac{x}{2}}\right)^2>\left(\frac{x}{2}\right)^2\Rightarrow e^x>\frac{x^2}{4}\Rightarrow\frac{e^x}{x}>\frac{x}{4}\Rightarrow(9)$
(10)	$\lim\limits_{x\to+\infty}\frac{Exp(x)}{x^n}=+\infty$	$e^{\frac{x}{n+1}}>\frac{x}{n+1}\Rightarrow\left(e^{\frac{x}{n+1}}\right)^{n+1}>\left(\frac{x}{n+1}\right)^{n+1}\Rightarrow e^x>\frac{x^{n+1}}{(n+1)^{n+1}}\Rightarrow\frac{e^x}{x^n}>\frac{x}{(n+1)^{n+1}}\Rightarrow(10)$
(11)	$\lim\limits_{x\to-\infty}x.Exp(x)=0^-$	$xe^x=-\frac{(-x)}{e^{(-x)}}$; $\frac{(-x)}{e^{(-x)}}\to0^+$ *when* $(-x)\to+\infty$ *then* (11)
(12)	$\lim\limits_{x\to-\infty}x^n\cdot Exp(x)=0^{\pm}$	$x^n\cdot e^x=(-1)^n\frac{(-x)^n}{e^{(-x)}}$; $(-x)\to+\infty$ $\frac{(-x)^n}{e^{(-x)}}\to0\Rightarrow(12)$
(13)	$\lim\limits_{x\to+\infty}\frac{\ln(x)}{x}=0^+$	$\frac{\ln(x)}{x}=\frac{\ln(x)}{e^{\ln(x)}}$; $\ln(x)\to+\infty\Rightarrow\frac{\ln(x)}{e^{\ln(x)}}\to0^+$
(14)	$\lim\limits_{x\to+\infty}\frac{(\ln(x))^n}{x}=0^+$	$\frac{(\ln(x))^n}{x}=\frac{(\ln(x))^n}{e^{\ln(x)}}=\frac{X^n}{e^X}$; $X=\ln(x)\to+\infty\therefore\frac{X^n}{e^X}\to0$
(15)	$\lim\limits_{x\to0^+}x.\ln(x)=0^-$	$x.\ln(x)=-\frac{\ln\left(\frac{1}{x}\right)}{\frac{1}{x}}$; $X=\frac{1}{x}\to+\infty$; $\frac{\ln X}{X}\to0^+\Rightarrow(15)$
(16)	$\lim\limits_{x\to0^+}x.(\ln(x))^n=0^-$	$x.(\ln(x))^n=(-1)^n\frac{\left[\ln\left(\frac{1}{x}\right)\right]^n}{\frac{1}{x}}=(-1)^n\frac{[\ln X]^n}{X}$; $X=\frac{1}{x}\to+\infty\therefore\frac{[\ln X]^n}{X}\to0$
(17)	$\lim\limits_{x\to+\infty}\frac{e^x}{\ln x}=+\infty$	$\frac{e^x}{\ln x}=\frac{e^x}{x}\cdot\frac{x}{\ln x}\to+\infty$
(18)	$\lim\limits_{x\to\pm\infty}x.\ln(1+\frac{1}{x})=1$	$x.\ln(1+\frac{1}{x})=\frac{\ln(1+\frac{1}{x})}{\frac{1}{x}}$; $X=\frac{1}{x}\to0\therefore\frac{\ln(1+X)}{X}\to1$
(19)	$\lim\limits_{x\to1}\frac{\ln(x)}{x-1}=1$	$\lim\limits_{x\to1}\frac{\ln(x)}{x-1}=\lim\limits_{x\to1}\frac{\ln(x)-\ln(1)}{x-1}=ln'(1)=\frac{1}{1}=1$
(20)	$\lim\limits_{n\to+\infty}\left(1+\frac{1}{n}\right)^n=e$	$\ln\left(1+\frac{1}{n}\right)^n=n\ln\left(1+\frac{1}{n}\right)=\frac{\ln\left(1+\frac{1}{n}\right)}{\frac{1}{n}}\to1\therefore\left(1+\frac{1}{n}\right)^n=e^{\ln\left(1+\frac{1}{n}\right)^n}\to e^1$

"In a product or a quotient, the Exponential function imposes its limit to the Power"
and ***"the Power function imposes its limt to the Logarithm function"***