Derivative of Composite functions

I – Review of the main formulas :

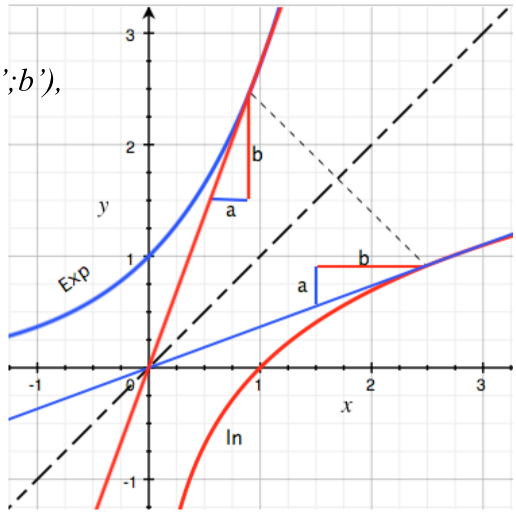
If u and v are two functions having derivatives u' and v' on the same Interval I =(a;b)then

$f =$	$u + v$	$k.u$	$u.v$	u^n $(n \in N)$	$\frac{1}{u}$ $(u \neq 0)$	$\frac{u}{v}$ $(v \neq 0)$	$\sqrt{u}$ $(u > 0)$	$Exp\ x$	$\ln x$ $(x > 0)$	$\sin x$
$f' =$	$u' + v'$	$k.u'$	$u'v + uv'$	$n.u^{n-1}u'$	$-\frac{u'}{u^2}$	$\frac{u'v - uv'}{v^2}$	$\frac{u'}{2\sqrt{u}}$	$Exp\ x$	$\frac{1}{x}$	$\cos x$

II – Composite functions formula :

- *Let the function u have a derivative u' on I = (a;b)*
- *Let f be a function having a derivative f' on J = u[<I>] = (a';b'),*
- *Let* $F = f \circ u$ *that is to say* $F(x) = f[u(x)]$,

Then $F'(x) = f'[u(x)].u'(x)$ $x \in I = (a;b)$



III – Derivative of the reciproqual function :

$f[f^{-1}(x)] = x$ => $\left(f^{-1}\right)'(x) = \dfrac{1}{f'\left[f^{-1}(x)\right]}$

IV – Applications :

1. **Derivative of *Exp[u(x)]* :** $[e^{u(x)}]' = e^{u(x)}\ u'(x)$
2. **Derivative of *ln[u(x)]* :** $(\ln[u(x)])' = \dfrac{u'(x)}{u(x)}$ *[with u(x) >0]*
3. **Derivative of f(x) = $u^k(x)$:** $[u^k(x)]' = k.\ u^{k-1}(x).\ u'(x)$ *for any* $k \neq -1$
4. **Derivative of *F(x) = $f(x)^{u(x)}$* ; *[f(x) >0]*** $F'(x) = [f(x)]^{u(x)}\left[u'(x).\ln[f(x)] + u(x).\dfrac{f'(x)}{f(x)}\right]$
5. **Derivative of *Sin[u(x)]* :** *sin'[u(x)] = u'(x).cos[u(x)]*

V – Exercises : Calculate the derivative of the following functions

(specify the Intervals of definition of these derivatives, then study the variations and graph the function)

(1) $f(x) = \ln\dfrac{x-1}{x+1}$; (2) $f(x) = \ln\dfrac{e^x - 1}{e^x + 1}$

(3) $f(x) = \dfrac{1}{\sqrt{x^2+1}}$; (4) $f(x) = \sqrt{\dfrac{x^2-1}{x^2+1}}$

(5) $f(x) = \left(1 - \dfrac{1}{x}\right)^{2x}$; (6) $f(x) = \left(1 + \dfrac{1}{x}\right)^{x^2}$

(7) $f(x) = \sin[\sin(\dfrac{\pi}{2} - x)]$; (8) $f(x) = \tan\dfrac{1}{x}$