Study of some Special Sequences

I. Let $u_{n+1} = \dfrac{2+u_n}{1+u_n}$ and $u_0 = 0$

1. Prove that for any n > 0, $u_n > 0$

2. Prove that for any $n \geq 0$ $u_{n+1} - \sqrt{2} = \dfrac{\sqrt{2} - u_n}{(1+u_n)\left(1+\sqrt{2}\right)}$

3. Derive from the above that for any $n \geq 0$ $\left|u_{n+1} - \sqrt{2}\right| \leq k\left|u_n - \sqrt{2}\right|$ *with* $k = \dfrac{1}{1+\sqrt{2}}$

4. Prove that for any $n \geq 0$ $\left|u_n - \sqrt{2}\right| \leq k^n\sqrt{2}$

5. Prove that the limit of (u_n) is $\sqrt{2}$

6. .Graph (u_n) on the back of this page.