Space Geometry : Parallelism & Orthogonality

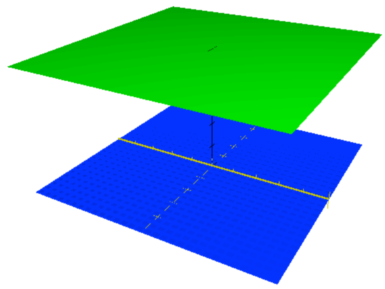
(non metric)

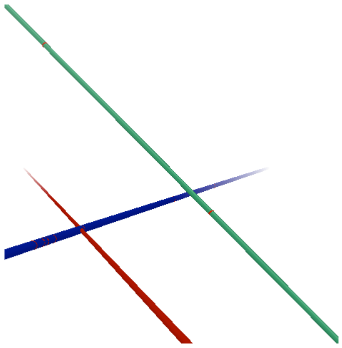
I. Paralell planes : [Theorem]

Two planes (P) and (P') are parallel

if and only if

(P) contains two <u>non parallel</u> lines (D) and (D'), respectively both parallel to (P').



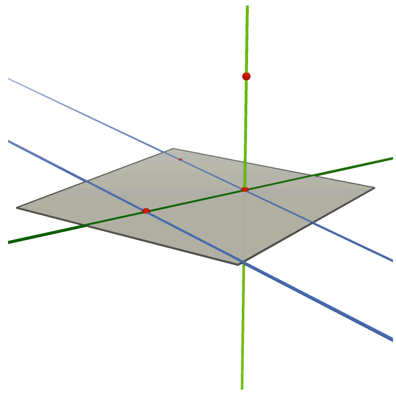


II. Orthogonality of two lines :

[Definition] Two lines (D) and (Δ), not coplanar, are orthogonal

if and only if

by a point A of (Δ) we can draw a line (D') paralell to (D) and perpendicular to (Δ).

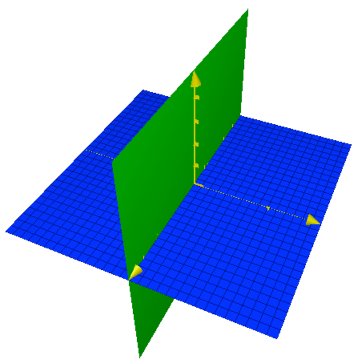


III. Line Perpendicular to a plane :

[Definition] a line (Δ) is perpendicular to a plane (P)

if and only if

(Δ) is orthogonal to (at least) two <u>non parallel</u> lines (D) and (D') of that plane (P).



IV. <u>Theorem</u> : if a line (Δ) is perpendicular to a plane (P) then it is orthogonal to <u>all</u> lines of that plane.

V. Perpendicular planes : [definition]

Two planes are perpendicular

if and only if

one contains a line perpendicular to the other plane

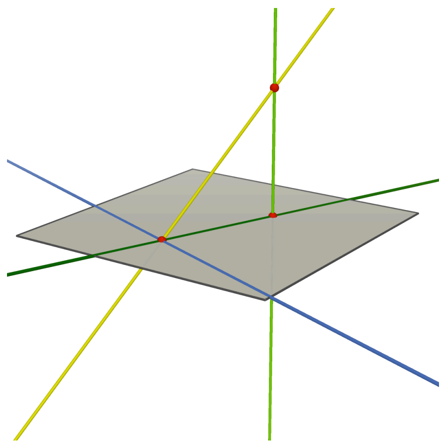
[That is a line orthogonal to two non parallel lines in that plane]

VI. Theorem of the 3 perpendiculars :

Let (D) be a line included in plane (P) and A ∈ (D).

From a point S out of the plane (P) we draw a line (Δ) perpendicular in I to the plane (P),

then if (IA) is perpendicular to (D) then (SA) is also perpendicular to (D).



<u>*Proof*</u> : $\begin{Bmatrix} (SI)\perp(P) \\ (IA)\perp(D) \end{Bmatrix} \Rightarrow \begin{Bmatrix} (D)\perp(IS) \\ (D)\perp(IA) \end{Bmatrix}$

$\Rightarrow (D)\perp(ISA) \Rightarrow (D)\perp(SA)$

Reciprocal : If (D) is perpendicular to (SA) and (SI) perpendicular to the plane (P) in I, then (D) is perpendicular to (IA).

<u>*Proof*</u> : $\begin{Bmatrix} (SI)\perp(P) \\ (SA)\perp(D) \end{Bmatrix} \Rightarrow \begin{Bmatrix} (D)\perp(SI) \\ (D)\perp(SA) \end{Bmatrix}$

$\Rightarrow (D)\perp(SAI) \Rightarrow (D)\perp(IA)$