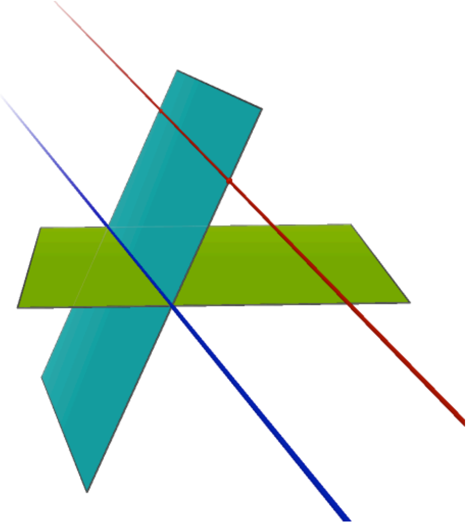
Space Geometry Basic Theorems

(non metric)

I. Preliminary theorems :



a. **Preliminary 1 :** Two planes cannot have one common point without containing a straigth line going through that point.

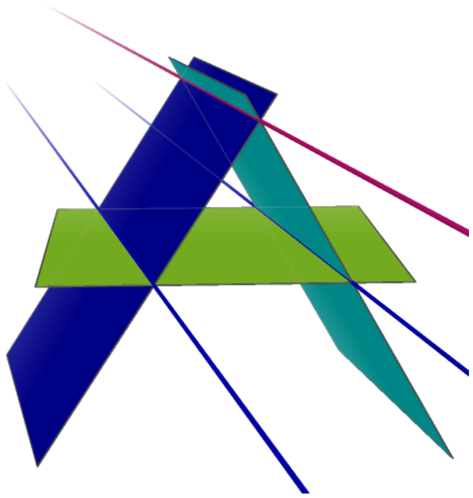
b. **Preliminary 2** : a straight line (D) is parallel to a plane (P) *if and only if* it is parallel to one line (D') included in (P).

c. **Preliminary 3 :** if a line (D) is parallel to a plane (P), then any plane (Q) containing (D) cuts (P) on (Δ) parallel to (D).

Proof : *Let (Δ) be the interception of (Q) and (P).*
Then the lines (D) and (Δ) are in the same plane (Q).
Therefore they are either parallel or intercepting.
But if (D) would cut (Δ) in a point I, then (D) would not be parallel to the plane (P). Hence (Δ) // (D).

II. Theorem of the roof

II. Theorem of the roof : (D) and (D') are two **parallel** lines generating the "*horizontal*" plane (P). Then if the two planes (Q) containing (D) and (Q') containing (D') intercept in one line (Δ), then (Δ) is parallel to the two lines (D) and (D') [and hence (Δ) // (P)].



Proof : *<u>from the preliminary theorems</u> :*

i. *(D) // (D') and (D') ⊂ (Q') ⇒ (D) // (Q') ;*
(D) ⊂ (Q) ⇒ (Q) ∩ (Q') = (Δ') // (D)

ii. *(D') // (D) and (D) ⊂ (Q) ⇒ (D') // (Q) ;*
(D) ⊂ (Q) ⇒ (Q) ∩ (Q') = (Δ") // (D')

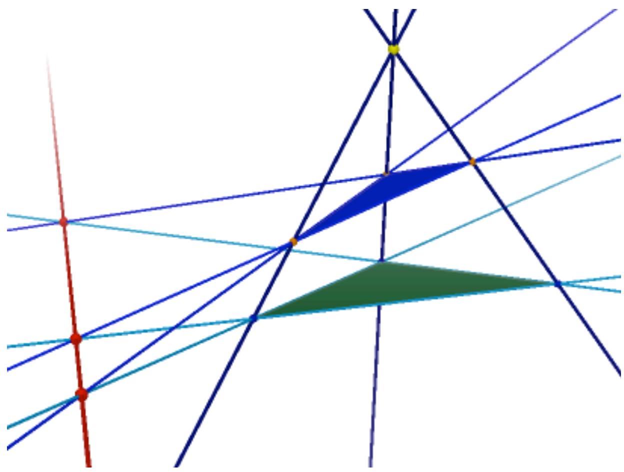
iii. *Let I ∈ (Q) ∩ (Q') , then I ∈ (Δ') and I ∈ (Δ").*

iv. *(Δ') // (D) and (D) // (D') then (Δ') // (D'),*
∴ (Δ') // (D') and (Δ") // (D')

v. *Hence (Δ') and (Δ") are one same line because **[From Euclid's axiom]** there is only one line going through I and // to (D'), therefore the interception line of the two planes (Q) and (Q') is parallel to the two lines (D) and (D') (and then to (P).*

III. DESARGUES Theorem

III. DESARGUES Theorem (XVIth): If (O,A,B,C) is a tetraedron (pyramid with 4 sides) of base ABC, and if A',B',C' are three points on the edges of that tetraedron, such that the plane (A'B'C') is <u>not parallel</u> to the plane (ABC).
Let U = (AB) ∩ (A'B'), V = (BC) ∩ (B'C'),
and W = (CA) ∩ (C'A').
Then the three points U,V,W are on a same line



Proof : *(too easy !) Let (Δ) be the interception line of the two planes (ABC) and (A'B'C'), then (Δ) contains all the common points of the two planes : specifically it contains U because U ∈ (ABC)) ∩ (A'B'C), and same for V and W.*