All that you ever wanted to know about Sequences without ever daring ask about them….

Definition : any list of Numbers written in a certain order is making a Numerical Sequence. We use a special notation to specify each term of the sequence in reference to it's rank in the list, this notation is generally U(n) or simply U_n. The **index** n shows the position of the number in the list. This index is a Natural number : 0, 1, 2 , 3, …, n-1, n, n+1,… The terms U_{n-1}, U_n and U_{n+1} are three following numbers in the sequence. U_0 represents the initial term of the sequence (index 0 = initial term). If the sequence is not defined for n = 0, the sequence starts with U_1.

Examples : *(1) Sequence of the numbers obtained by counting 3 by 3 from -5 :*
(-5, -2, 1, 4, 7, ...,) the general term of this sequence is $U_n = -5 + 3.n$
The first term is $U_0 = -5$ *; the 11*th *term is* $U_{10} = -5 + 3 \times 10 = 25$ *; 100*th *term* $U_{99} = -5 + 3 \times 99 = 292$.

(2) Sequence of the numbers obtained by multiplying every previous number by 2 starting with 3 :
(3, 6, 12, 24, 48, ...) The general term of that sequence is $V_n = 3.(2)^n$
First term : $V_0 = 3$ *; 10*th *term :* $V_9 = 3.(2)^9 = 3 \times 512 = 1536$ *;* $V_{20} = 3.(2)^{20} = 3 \times 1024^2 = 3\ 145\ 728$.

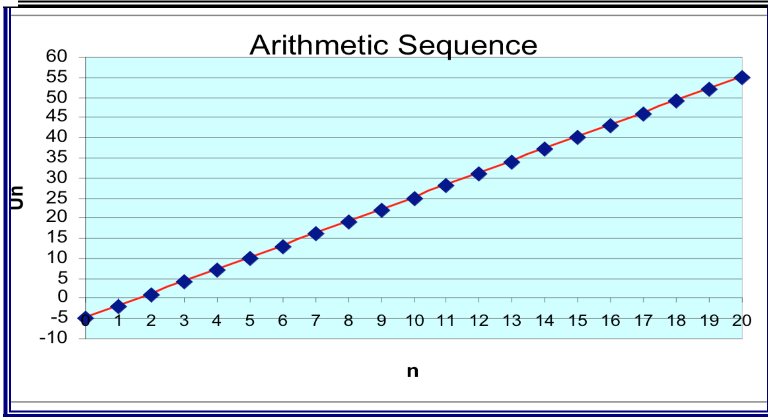
(3) Sequence of the decimals digits of the number π : (3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9 …..) In this sequence the 10th term (rank n = 9) is 5, but there is no simple formula to find it …

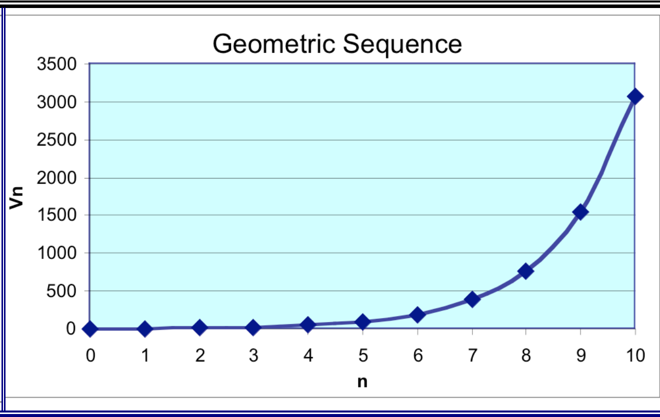
(4) Fibonacci sequence : (1, 1 , 2, 3, 5, 8, 13, 21, 34, …) in this sequence every term is the sum of the two previous ones. We can get as many terms as we want but we cannot get the value of the 100th term without calculating the 99 terms before it, We have : $U_{n+1} = U_n + U_{n-1}$, but the formula that would provide the direct value is complicated (see Binet formula on the website)

(5) Sequencce of the squares of the integers : (0, 1, 4, 9, 16, 25, 36, …) It's easy to find that $U_n = n^2$ and that $U_{100} = 100^2 = 10\ 000$.

The type (1) sequences are named **Arithmetic Sequences,**
The type (2) sequences are named **Geometric Sequences,**
The sequences (3) , (4) , (5) are neither arithmetic or geometric.

ARITHMETIC Sequence	**GEOMETRIC Sequence**
Definitions 1	
Every term is obtained by adding the same number **r** to the previous one (**r** = « reason ») $\mathbf{U_{n+1} = U_n + r}$	Every term is obtained by multiplying each term by the same number **q** (**q** = « quotient») $\mathbf{V_{n+1} = q.V_n}$
Definitions 2	
The **difference** of two following terms is **constant** $\mathbf{U_{n+1} - U_n = r}$	The **quotient** of two following terms is **constant** $\frac{V_{n+1}}{V_n} = q$
General Formulas	
$\mathbf{U_n = a + n.r}$ 1st term : $\mathbf{U_0 = a}$; reason = **r** (*ratio* = difference)	$\mathbf{V_n = a.q^n}$ 1st term : $\mathbf{V_0 = a}$; reason = **q** (quotient = *ratio*)
Characteristic property # 1	
Every term is the arithmetic mean of the terms which are équidistants from it. $U_n = \frac{U_{n-p} + U_{n+p}}{2}$	Every term is the quadratic (or geometric) mean of the terms which are équidistants from it. $V_n = \sqrt{V_{n-p} . V_{n+p}}$
Characteristic property # 2	
Variation of **Linear** type	Variation of **Exponential** type


Arithmetic Sequence
Un
n
60
55
50
45
40
35
30
25
20
15
10
5
0
-5
-10
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20


Geometric Sequence
Vn
n
3500
3000
2500
2000
1500
1000
500
0
0 1 2 3 4 5 6 7 8 9 10