## Geometric Series in Geometry (2) Von Koch's snow flake

## I. Construction of the basic pattern :

1. Divide the sides of an equilateral triangle into three equal parts
2. Erase the middle part of each side and replace it by an equilateral triangle
3. Do the same on each of the 4 new sides of the side ...

$\mathrm{n}=0$

$\mathrm{n}=1$

$\mathrm{n}=2$

$\mathrm{n}=3$
II. A paradoxal situation : we want to show that the length of the curve goes to infinity, while the area inside has a finite limit.
i. Let's call $\mathrm{P}_{\mathrm{n}}$ the length of the perimeter of the $\mathrm{n}^{\text {th }}$ picture
ii. Let's call $\mathrm{C}_{\mathrm{n}}$ the number of sides
iii. Let's call $\mathrm{A}_{\mathrm{n}}$ the area enclosed in the nth picture.
4. Write $\mathrm{P}_{0}, \mathrm{C}_{0}, \mathrm{~A}_{0}$ (the length of the side of the initial triangle is 1 unit)
5. Write $\mathrm{P}_{1}, \mathrm{C}_{1}, \mathrm{~A}_{1}$, then $\mathrm{P}_{2}, \mathrm{C}_{2}, \mathrm{~A}_{2}$ :
6. Write the relationship between $P_{n+1}$ and $P_{n} ; C_{n+1}$ and $C_{n} ; A_{n+1}$ and $A_{n}$
7. Find the limits of each sequence $\left(\mathrm{P}_{\mathrm{n}}\right),\left(\mathrm{C}_{\mathrm{n}}\right),\left(\mathrm{A}_{\mathrm{n}}\right)$.
