Special Series in Geometry (3)
Rieman's Integral's sum

0. Pove the following formulas :

$$(n+1)^2 = 1 + 2\sum_{i=1}^{n} i + n \qquad \text{and} \qquad (n+1)^3 = 1 + 3\sum_{i=1}^{n} i^2 + 3\sum_{i=1}^{n} i + n$$

$$S_n = 1+2+3+...+n = \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \text{and} \qquad T_n = 1^2+2^2+3^2+...+n^2 = \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

1. Construction of the basic patterns :

Fig.1 : y = x, n = 10

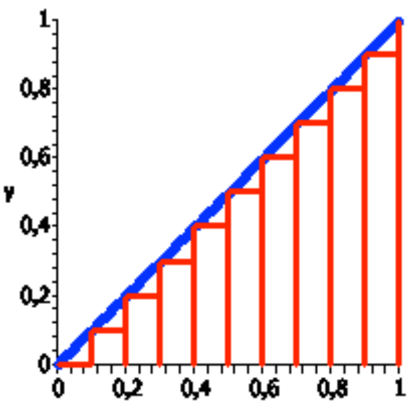
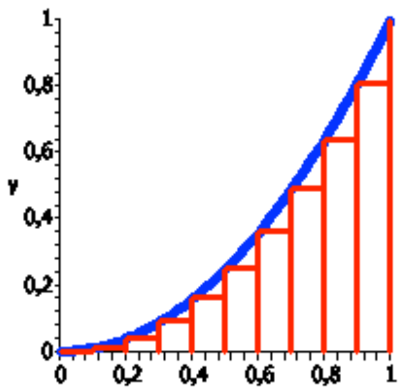


Fig.2 : y = x^2 , n = 10



In these two graphics the Interval [0,1] is divided into n = 10 equal parts.

2. Evaluate the area of the rectangles defined by this division into 10 parts :

$$A_{10} = \sum_{i=0}^{i=9} \frac{1}{10}\left(\frac{i}{10}\right) \qquad B_{10} = \sum_{i=0}^{i=9} \frac{1}{10}\left(\frac{i}{10}\right)^2$$

3. Evaluate the area of the rectangles defined by a division into n equal parts :

$$A_n = \frac{1}{n^2}\sum_{i=0}^{i=n-1} i \qquad B_n = \frac{1}{n^3}\sum_{i=0}^{i=n-1} i^2$$

4. Find the limits A and B of (A_n) and (B_n). *Area under the curve on [0;1]:* $S = \sum_{i=0}^{\infty} \frac{1}{n} f\left(\frac{i}{n}\right)$