

### Study of a mixed sequence

*non recursive ! non geometric ! non arithmetic !*

$$u_n = \left(1 + \frac{2}{n}\right)u_{n-1} + \frac{6}{n} \text{ and } u_0 = 5$$

I. Calculate  $u_1 ; u_2 ; u_3 ; u_4 ; u_5$

II. Let  $d_n = u_{n+1} - u_n$ .

1. Calculate  $d_0 , d_1 , d_2 , d_3 , d_4$ .

2. Check that the five first terms of  $(d_n)$  form an Arithmetic sequence :

Give the first term  $d_0$ , and the reason  $r$  :

III. Prove by recurrence that for any  $n \geq 0$ ,  $u_n = 4n^2 + 12n + 5$ .

IV. Prove that  $(d_n)$  is actually an arithmetic sequence.

1. Find the expression of the sum  $S_n$  of the  $n^{\text{th}}$  first terms of  $(d_n)$  :

$$S_n = d_0 + d_1 + d_2 + \dots + d_{n-1}$$

2. Give the relationship between  $S_n$  and  $u_n$ . What is the limit of  $(u_n)$  ?