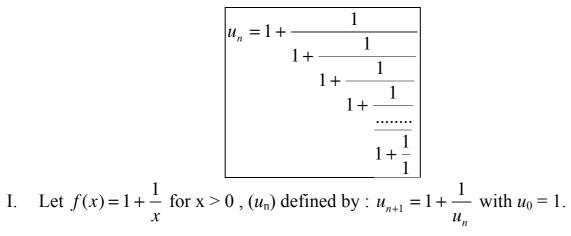
Problems of Convergence of Non Monotonous Recursive Sequences using Recurrence Reasonning :

Study of the sequence defined by (*n* consecutive fractions)



- 1. Write u_1 ; u_2 ; u_3 .
- 2. On the back of the page graph the function f on [0; 2] and show the construction of the first terms of the sequence (u_n) .
- 3. Show by <u>recurrence</u>, that for any $n \ge 0$, $1 \le u_n \le 2$
- 4. From the graph indicate whether (u_n) is monotonous.
- 5. From the graph indicate whether (u_n) is converging. If yes, what is the limit?

II. Let $g = f \circ f \Leftrightarrow g(x) = f[f(x)]$

- 1. Calculate g(x), and show that g is increasing on $[0; +\infty)$
- 2. Let $v_n = u_{2n}$ and $w_n = u_{2n+1}$
 - i. Show that $v_{n+1} = g(v_n)$ and that $w_{n+1} = g(w_n)$; give v_0 and w_0
 - ii. Show <u>by recurrence</u>, that (w_n) is decreasing and that (v_n) is increasing.
 - iii. Prove that for any $n \ge 0 |v_{n+1} w_{n+1}| \le \frac{1}{4} |v_n w_n|$
 - iv. Prove <u>by recurrence</u> that for any $n \ge 0$ $|v_n w_n| \le \left(\frac{1}{4}\right)^n$,
 - v. Prove that $\lim |v_n w_n| = 0$
 - vi. The two sequences (v_n) and (w_n) are <u>adjacent</u>, then have the same limit α . Prove that this limit is the positive solution of the equation $x = 1 + \frac{1}{x}$, and give the exact value of α .