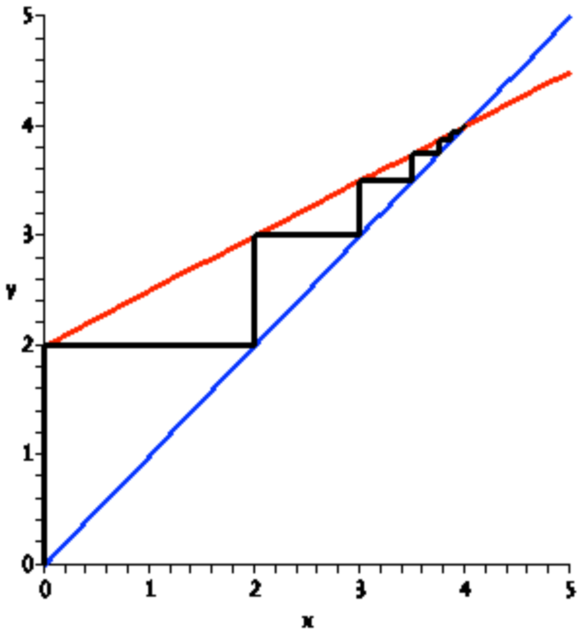
# Numerical Sequences (1.1)

Let *f* be the function defined by $f(x) = \frac{1}{2}x + 2$ for x ≥ 0.

Study of the sequence $(u_n)$ defined by $u_{n+1} = f(u_n) = \frac{1}{2}u_n + 2$ ; n ≥1 and $u_0 = 0$.

1. Graph the function f on [0 ; +∞ [ and draw the first terms of the sequence $(u_n)$.
   Find the coordinates of the intersection of (Cf) with the first bisector (y=x)
   Indicate from the graph whether or not the sequence is :
   i. Monotonous (if yes how) :
   ii. Bounded (*if yes, what are the boundaries ?)*
   iii. Does-it seam to have a limit (*if yes which one is it?*)?
2. Let $v_n = u_n - 4$ *for any n>0.*
   Show that the new sequence $(v_n)$ is a geometric sequence :
   1. Find its first term and its reason.
   2. Find the expression of $v_n$ directly in function of n.
   3. Deduct the limit of $v_n$.
   4. Find the expression of $u_n$ in function *of* $v_n$
   5. Find the limit of $u_n$
   6. Check the *result on your graph.*




1) *From the graph of the sequence we can tell that the sequence $(v_n)$ is monotonous, bounded by 0 and 4 and seems to be converging towards 4 (abscissa of the interception of f with the 1st bisector.*
2) *$(v_n)$ is a geometric sequence because the ratio of two consecutive terms is constant :*

$$v_{n+1} = u_{n+1} - 4 = \frac{1}{2}u_n + 2 - 4 == \frac{1}{2}u_n - 2 = \frac{1}{2}(u_n - 4) = \frac{1}{2}v_n;\ v_0 = u_0 - 4 = -4 \text{ and the reason } q = \frac{1}{2}$$

$$\therefore v_n = -4\left(\frac{1}{2}\right)^n \Rightarrow \lim_{n\to+\infty} v_n = 0\ (\because\ |q| < 1)$$

$$v_n = u_n - 4 \Leftrightarrow u_n = v_n + 4 \text{ then } \lim_{n\to+\infty} u_n = 0 + 4 = 4$$

The interception point of the graph of f and of the 1st bisector (y=x) yields the equation

$$\frac{1}{2}x + 2 = x \text{ and } x > 0 \Leftrightarrow x = 4$$

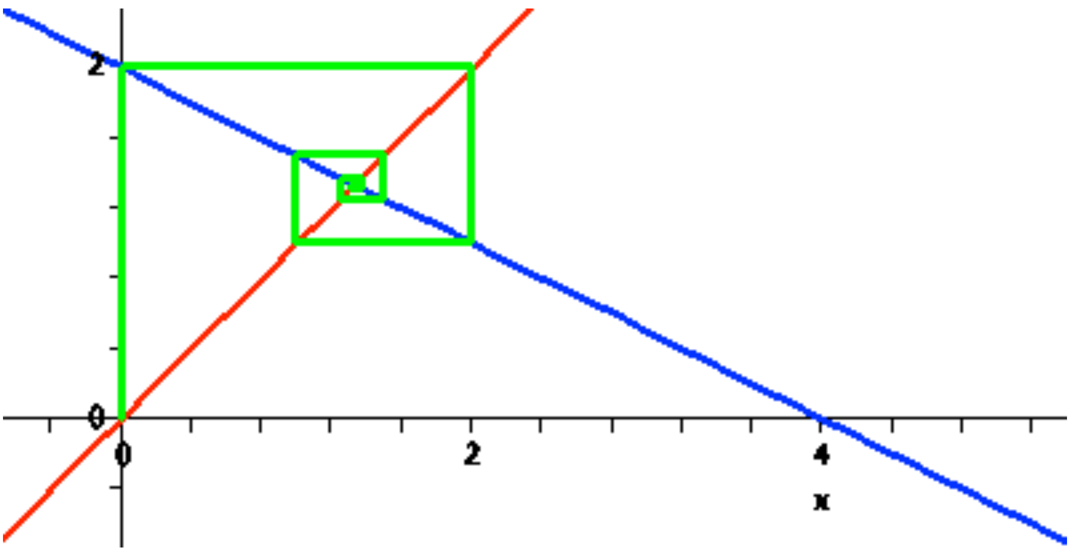
# Numerical Sequences (1.2)

Let $f$ be the function defined by $f(x) = -\frac{1}{2}x + 2$ for x ≥ 0.

Study of the sequence $(u_n)$ defined by $u_{n+1} = f(u_n) = -\frac{1}{2}u_n + 2$ ; n ≥1 and $u_0 = 0$.

3. Graph the function f on [0 ; +∞ [ and draw the first terms of the sequence $(u_n)$.
   Find the coordinates of the intersection of (Cf) with the first bisector (y=x)
   Indicate from the graph whether or not the sequence is :
   i. Monotonous (if yes how) :
   ii. Bounded (*if yes, what are the boundaries ?)*
   iii. Does-it seam to have a limit (*if yes which one is it?*)?

4. Let $v_n = u_n - \frac{4}{3}$ *for any n>0.*
   Show that the new sequence $(v_n)$ is a geometric sequence :
   1. Find its first term and its reason.
   2. Find the expression of $v_n$ directly in function of n.
   3. Deduct the limit of $v_n$.
   4. Find the expression of $u_n$ in function *of* $v_n$
   5. Find the limit of $u_n$
   6. Check the *result on your graph.*




3) *From the graph of the sequence we can tell that the sequence $(v_n)$ is not monotonous, bounded by 0 and 2 and seems to be converging towards 1.3 (abscissa of the interception of f with the 1st bisector.*

4) *$(v_n)$ is a geometric sequence because the ratio of two consecutive terms is constant :*

$$v_{n+1} = u_{n+1} - \frac{4}{3} = -\frac{1}{2}u_n + 2 - \frac{4}{3} == -\frac{1}{2}u_n + \frac{2}{3} = -\frac{1}{2}(u_n - \frac{4}{3}) = -\frac{1}{2}v_n ; v_0 = u_0 - \frac{4}{3} = -\frac{4}{3} \text{ and the reason } q = -\frac{1}{2}$$

$$\therefore v_n = -\frac{4}{3}\left(-\frac{1}{2}\right)^n \Rightarrow \lim_{n\to+\infty} v_n = 0 \ (\because \ |q| < 1)$$

$$v_n = u_n - \frac{4}{3} \Leftrightarrow u_n = v_n + \frac{4}{3} \text{ then } \lim_{n\to+\infty} u_n = 0 + \frac{4}{3} = \frac{4}{3}$$

The interception point of the graph of f and of the 1st bisector (y=x) yields the equation

$$-\frac{1}{2}x + 2 = x \text{ and } x > 0 \Leftrightarrow 3x = 4 \text{ and } x > 0 \Leftrightarrow x = \frac{4}{3}$$