Fibonacci's Numerical Sequence (2.5)

1 ; 1 ; 2 ; 3 ; 5 ; 8 ; 13 ; 21 ; … known as the Fibonacci's (XIII[th] Century)

where $F_1 = 1$; $F_2 = 1$; $F_3 = F_1 + F_2 = 2$; … ; $\mathbf{F_{n+1} = F_n + F_{n-1}}$

1. **Construction of the Fibonacci's sequence : Rabbits generation** :

Here is a famous problem posed in the XIII[th] century by Leonardo de Pisano , better known as Fibonacci: suppose we have one pair of newborn rabbits of both genders. We assume that the following conditions are true :

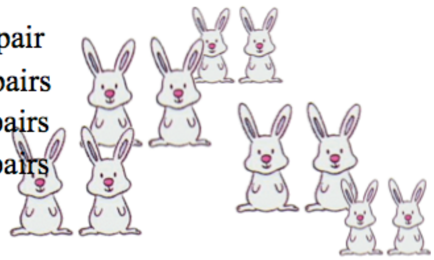
a. It takes a newborn rabbit one month to become an adult.

b. A pair of adult rabbits of both genders will produce one pair of newborn rabbits of both genders each month, beginning one month after becoming adults.

c. The rabbits do not die

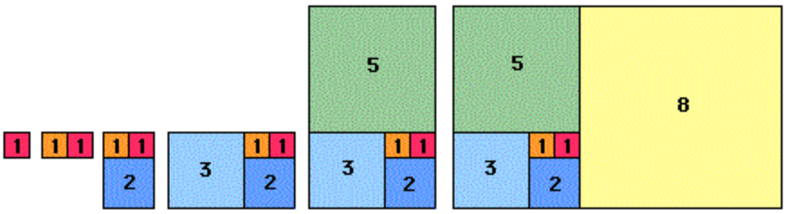
How many rabbits will there be one year later?

1st month	1 pair
2nd month	1 pair
3rd month	2 pairs
4th month	3 pairs
5th month	5 pairs



2. **Construction of the Fibonacc'is sequence with Squares** :

we start with two small squares of size 1 next to each other. On top of these draw a square of size 2. Then draw a new square of size 3 just as the picture shows, and the square of size 5, size 8, size 13…



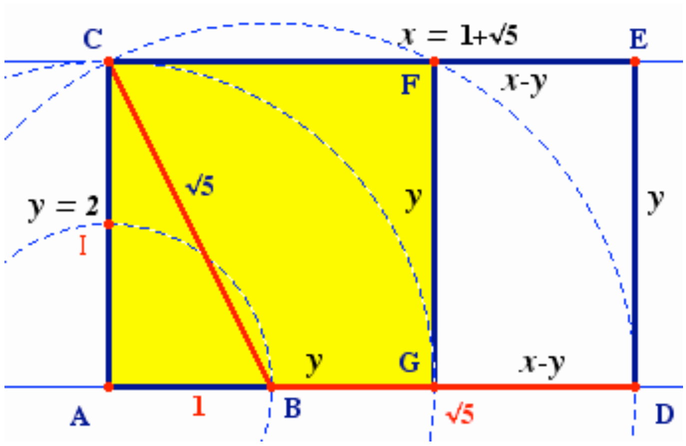
FIBONACCI SQUARES

3. **Relationship between the Fibonacci's numbers and the Golden Number**

In this rectangle the ratio between the two sides of the rectangle is such that its equal to that of the small rectangle inside it :

$$\varphi = \frac{x}{y} = \frac{y}{x-y}$$

Find the value of φ, and then construct a rectangle according to this ratio.



Solution :

$$\varphi = \frac{x}{y} = \frac{y}{x-y} = \frac{1}{\frac{x}{y}-1} = \frac{1}{\varphi - 1}$$

$$\therefore \varphi = \frac{1}{\varphi-1} \Leftrightarrow \varphi^2 - \varphi - 1 = 0$$

$$\therefore \qquad \varphi = \frac{1+\sqrt{5}}{2} \quad (\varphi > 0)$$

Note : The golden number is equal to the limit of the ratio of $\frac{F_{n+1}}{F_n}$…

Let $U_n = \frac{F_{n+1}}{F_n} = \frac{F_n + F_{n-1}}{F_n} = 1 + \frac{F_{n-1}}{F_n} = 1 + \frac{1}{U_{n-1}}$; $U_1 = 1$…*Hence one can prove that* U_n *has a finite positive limit x, then* U_{n-1} *has the same limit x then we mut have* $x = 1 + \frac{1}{x} \quad \therefore \quad x = \varphi$