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Numerical Sequences (2.3)

Problem : Let f be the function defined by $f(x)=\dfrac{-7x-8}{2x+1}$.

Study of the sequence (v_n) defined by $u_{n+1}=f(u_n)=\dfrac{-7u_n-8}{2u_n+1}$; n ≥1 and u_0 = - 0.8

1. Graph the function f on [-6 ; +6] and build the first terms of the sequence (u_n).
2. Find the coordinates of the intersection of (Cf) with the first bisector (y=x)
3. Indicate from the graph whether or not the sequence is :
   i. Monotonous (if yes how) :
   ii. Bounded (*if yes, what are the boundaries ?)*
   iii. Does it seem to have a limit (*if yes which one is it?*)?
4. Let $v_n=\dfrac{2u_n+1}{u_n+2}$ *for any* $n\ge 0$. Show that the new sequence (v_n) is an **arithmetic sequence**
   (i) Find its first term v_0 and its reason r
   (ii) Find the expression of v_n directly in function of n.
   (iii) Deduct the limit of v_n.
   (iv) Find the expression of u_n in function *of* v_n
   (v) Deduct the limit of u_n
   (vi) Check the *result on your graph*

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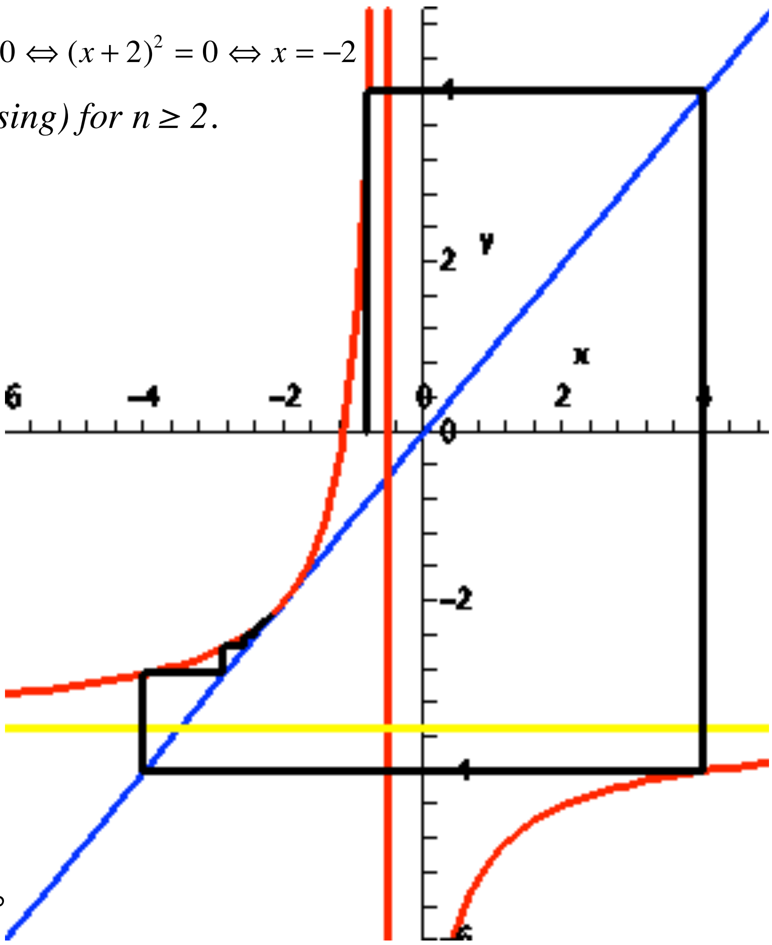
*1-2 **Intersection** of (C_f) with the 1st **Bisector** (y=x) :*

$$\frac{-7x-8}{2x+1}=x \Leftrightarrow 2x^2+8x+8=0 \Leftrightarrow x^2+4x+4=0 \Leftrightarrow (x+2)^2=0 \Leftrightarrow x=-2$$

3- The sequence (u_n) *is **monotonous** (increasing) for* $n\ge 2$.

and for any $n\ge 0$, $-4\le u_n\le 4$.

It seems to be converging towards -2.



4- $v_n=\dfrac{2u_n+1}{u_n+2}$ *; then we get :*

$$v_{n+1}=\frac{2u_{n+1}+1}{u_{n+1}+2}=\frac{2\dfrac{-7u_n-8}{2u_n+1}+1}{\dfrac{-7u_n-8}{2u_n+1}+2}$$

$$\therefore v_{n+1}=\frac{-12u_n-15}{-3u_n-6}=\frac{4u_n+5}{u_n+2}=2+\frac{2u_n+1}{u_n+2}$$

$\therefore$ $v_{n+1}=2+v_n$ *Arithmetic sequence*

Reason of (v_n) : $r=2$;

$$1^{st}\ \textit{term} : v_0=\frac{2u_0+1}{u_0+2}=-\frac{0.6}{3.6}=-\frac{1}{6}$$

$$\therefore \textit{for any } n\ge 0,\ v_n=-\frac{1}{6}+2n \therefore \lim v_n=+\infty$$

$$v_n=\frac{2u_n+1}{u_n+2}\Leftrightarrow u_n=\frac{-2v_n+1}{v_n-2}=-2-\frac{5}{v_n-2} \quad \therefore \quad \lim u_n=-2$$ [*Interception of* (*Cf*) *with* (Δ)].