

## Numerical Sequences (2.3)

Problem : Let  $f$  be the function defined by  $f(x) = \frac{-7x - 8}{2x + 1}$ .

Study of the sequence  $(v_n)$  defined by  $u_{n+1} = f(u_n) = \frac{-7u_n - 8}{2u_n + 1}$  ;  $n \geq 1$  and  $u_0 = -0.8$

1. Graph the function  $f$  on  $[-6 ; +6]$  and build the first terms of the sequence  $(u_n)$ .
2. Find the coordinates of the intersection of  $(C_f)$  with the first bisector ( $y=x$ )
3. Indicate from the graph whether or not the sequence is :
  - i. Monotonous (if yes how) :
  - ii. Bounded (if yes, what are the boundaries ?)
  - iii. Does it seem to have a limit (if yes which one is it)?
4. Let  $v_n = \frac{2u_n + 1}{u_n + 2}$  for any  $n \geq 0$ .

Show that the new sequence  $(v_n)$  is an **arithmetic sequence** : [use back of page]

- (i) Find its first term  $v_0$  and its reason  $r$
- (ii) Find the expression of  $v_n$  directly in function of  $n$ .
- (iii) Deduct the limit of  $v_n$ .
- (iv) Find the expression of  $u_n$  in function of  $v_n$
- (v) Deduct the limit of  $u_n$
- (vi) Check the *result on your graph*.

