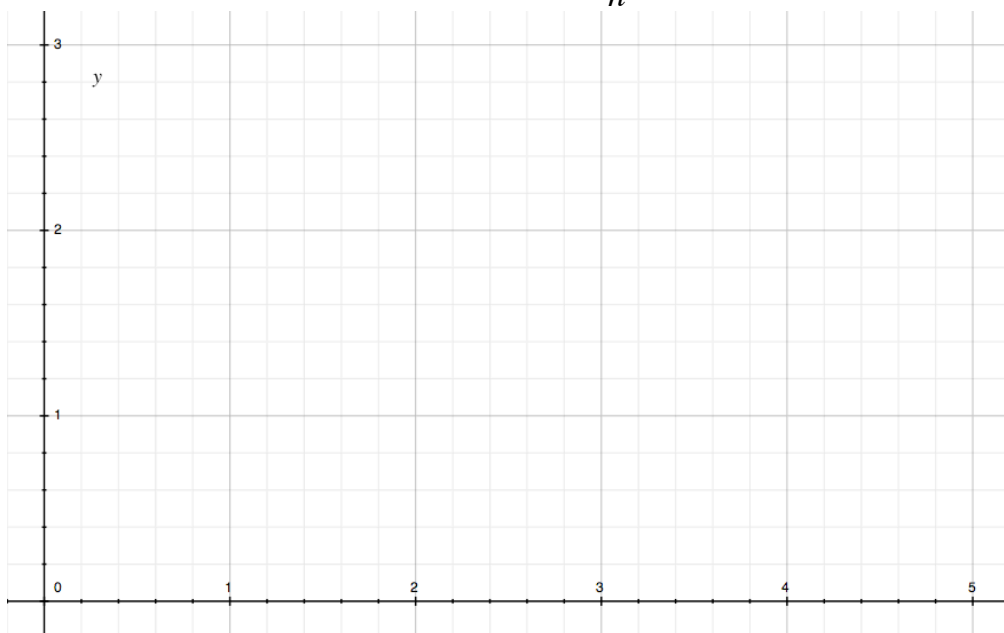


## Numerical Sequences (2.1)

Problem I : Let  $f$  be the function defined by  $f(x) = \frac{2x+3}{x+4}$  for  $x \geq 0$ .

Study the Sequence defined by the formula  $u_n = f(n) = \frac{2n+3}{n+4}$  for every  $n \in \mathbb{N}$ .

- a. Graph the function  $f$  on  $[0 ; +\infty [$  and draw the first terms of the sequence  $(u_n)$ .  
 Indicate from the graph whether or not the sequence is :
  - i. Monotonous (if yes how) :
  - ii. Bounded (if yes, what are the boundaries ?)
  - iii. Does-it seem to have a limit (if yes which one is it?)?
- b. Prove that  $(u_n)$  is increasing
- c. Prove that  $(u_n)$  is bounded by 0 and 2.
- d. Find for which value of  $n$  we have :  $2 - \varepsilon < u_n < 2$  with  $\varepsilon = 10^{-2}$
- e. Prove that for any  $n \geq 1$  we have  $|u_n - 2| \leq \frac{5}{n}$ . Conclusion ?



1.