

**All that you ever wanted to know about Sequences without ever daring ask about them....**

**Definition :** any list of Numbers written in a certain order is making a Numerical Sequence. We use a special notation to specify each term of the sequence in reference to it's rank in the list, this notation is generally  $U(n)$  or simply  $U_n$ . The index  $n$  shows the position of the number in the list. This index is a Natural number:  $0, 1, 2, 3, \dots, n-1, n, n+1, \dots$ . The terms  $U_{n-1}, U_n$  and  $U_{n+1}$  are three following numbers in the sequence.  $U_0$  represents the initial term of the sequence (rank  $0 = 1$ st term).

**Examples :** (1) Sequence of the numbers obtained by counting 3 by 3 from -5 :

$(-5, -2, 1, 4, 7, \dots)$  the general term of this sequence is  $U_n = -5 + 3n$

The first term is  $U_0 = -5$  ; the 11<sup>th</sup> term is  $U_{10} = -5 + 3 \times 10 = 25$  ; 100<sup>th</sup> term  $U_{99} = -5 + 3 \times 99 = 292$ .

(2) Sequence of the numbers obtained by multiplying every previous number by 2 starting with 3 :

$(3, 6, 12, 24, 48, \dots)$  The general term of that sequence is  $V_n = 3 \cdot (2)^n$

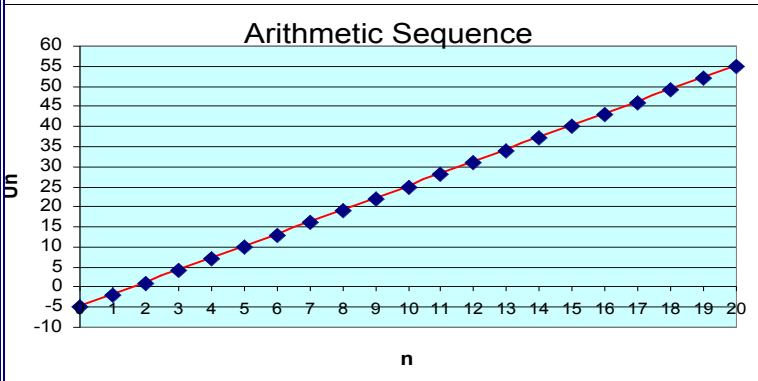
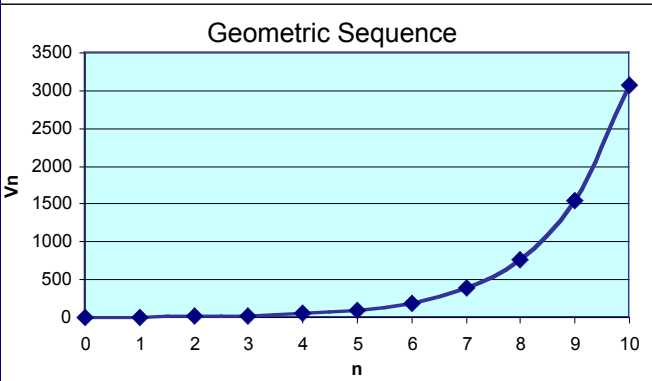
First term :  $V_0 = 3$  ; 10<sup>th</sup> term :  $V_9 = 3 \cdot (2)^9 = 3 \times 512 = 1536$  ;  $V_{20} = 3 \cdot (2)^{20} = 3 \times 1024^2 = 3 \times 145 \ 728$ .

(3) Sequence of the decimals digits of the number  $\pi$  :  $(3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9 \dots)$ . In this sequence the 10th term (rank  $n = 9$ ) is 5, but there is no simple formula to find it ...

(4) Fibonacci sequence :  $(1, 1, 2, 3, 5, 8, 13, 21, 34, \dots)$  in this sequence every term is the sum of the two previous ones. We can get as many terms as we want but we cannot get the value of the 100<sup>th</sup> term without calculating the 99 terms before it, We have :  $U_{n+1} = U_n + U_{n-1}$ . But the formula that would provide the direct value is complicated (see Binet formula on the website)

(5) Sequence of the squares of the integers :  $(0, 1, 4, 9, 16, 25, 36, \dots)$  It's easy to find that  $U_n = n^2$  and that  $U_{100} = 100^2 = 10 \ 000$ .

The type (1) sequences are named **Arithmetic Sequences**,  
 The type (2) sequences are named **Geometric Sequences**,  
 The sequences (3) , (4) , (5) are neither arithmetic or geometric.

<b>ARITHMETIC Sequence</b>	<b>GEOMETRIC Sequence</b>
<b>Definitions 1</b>	
Every term is obtained by <b>adding</b> the same number <b>r</b> to the previous one ( <b>r</b> = « reason »)  $U_{n+1} = U_n + r$	Every term is obtained by <b>multiplying</b> each term by the same number <b>q</b> ( <b>q</b> = « quotient »)  $V_{n+1} = q \cdot V_n$
<b>Definitions 2</b>	
The <b>difference</b> of two following terms is <b>constant</b>  $U_{n+1} - U_n = r$	The <b>quotient</b> of two following terms is <b>constant</b>  $\frac{V_{n+1}}{V_n} = q$
<b>General Formulas</b>	
$U_n = a + n \cdot r$ 1 <sup>st</sup> term : $U_0 = a$ ; reason = <b>r</b> ( <i>ratio</i> = difference)	$V_n = a \cdot q^n$ 1 <sup>st</sup> term : $V_0 = a$ ; reason = <b>q</b> ( <i>quotient</i> = <i>ratio</i> )
<b>Characteristic property # 1</b>	
Every term is the <b>arithmetic mean</b> of the terms which are équidistants from it. $U_n = \frac{U_{n-p} + U_{n+p}}{2}$	Every term is the <b>quadratic mean</b> of the terms which are équidistants from it. $U_n = \sqrt{U_{n-p} \cdot U_{n+p}}$
<b>Characteristic property # 2</b>	
Variation of <b>Linear</b> type	Variation of <b>Exponential</b> type
 <p style="text-align: center;">Arithmetic Sequence</p>	 <p style="text-align: center;">Geometric Sequence</p>