## *All that you ever wanted to know about Sequences without ever daring ask about them….*

**Definition** : any list of Numbers written in a certain order is making a Numerical Sequence. We use a special notation to specify each term of the sequence in reference to it's rank in the list, this notation is generally U(n) or simply $U_n$. The index n shows the position of the number in the list. This index is a Natural number : 0, 1, 2 , 3, …, n-1, n, n+1,… The terms $U_{n-1}$, $U_n$ and $U_{n+1}$ are three following numbers in the sequence. $U_0$ represents the initial term of the sequence (rank 0 = 1st term).

***Examples*** : *(1) Sequence of the numbers obtained by counting 3 by 3 from -5 :*

*(-5, -2, 1, 4, 7, …, ) the general term of this sequence is* $U_n = -5 + 3.n$

*The first term is* $U_0 = -5$ *; the* $11^{th}$ *term is* $U_{10} = -5 + 3 \times 10 = 25$ *;* $100^{th}$ *term* $U_{99} = -5 + 3 \times 99 = 292$.

*(2) Sequence of the numbers obtained by multiplying every previous number by 2 starting with 3 :*

*(3, 6, 12, 24, 48, …) The general term of that sequence is* $Vn = 3.(2)^n$

*First term :* $V_0 = 3$ *;* $10^{th}$ *term :* $V_9 = 3.(2)^9 = 3 \times 512 = 1536$ *;* $V_{20} = 3.(2)^{20} = 3 \times 1024^2 = 3\ 145\ 728$.

(3) Sequence of the decimals digits of the number **π** : (3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9 …..) In this sequence the 10th term (rank n = 9) is 5, but there is no simple formula to find it …

(4) Fibonacci sequence : (1, 1 , 2, 3, 5, 8, 13, 21, 34, …) in this sequence every term is the sum of the two previous ones. We can get as many terms as we want but we cannot get the value of the $100^{th}$ term without calculating the 99 terms before it, We have : $U_{n+1} = U_n + U_{n-1}$, But the formula that would provide the direct value is complicated (see Binet formula on the website)

(5) Sequencce of the squares of the integers : (0, 1, 4, 9, 16, 25, 36, …) It's easy to find that $U_n = n^2$ and that $U_{100} = 100^2 = 10\ 000$.

The type (1) sequences are named **Arithmetic Sequences,**
The type (2) sequences are named **Geometric Sequences,**
The sequences (3) , (4) , (5) are neither arithmetic or geometric.

| **ARITHMETIC Sequence** | **GEOMETRIC Sequence** |
|---|---|
| **Definitions 1** | |
| Every term is obtained by adding the same number **r** to the previous one (**r** = « reason ») $U_{n+1} = U_n + r$ | Every term is obtained by multiplying each term by the same number **q** (**q** = « quotient») $V_{n+1} = q.V_n$ |
| **Definitions 2** | |
| The **difference** of two following terms is **constant** $U_{n+1} - U_n = r$ | The **quotient** of two following terms is **constant** $\frac{V_{n+1}}{V_n} = q$ |
| **General Formulas** | |
| $U_n = a + n.r$ — $1^{st}$ term : $U_0 = a$ ; reason = **r** (*ratio* = difference) | $V_n = a.q^n$ — $1^{st}$ term : $V_0 = a$ ; reason = **q** (quotient = *ratio*) |
| **Characteristic property # 1** | |
| Every term is the arithmetic mean of the terms which are équidistants from it. $U_n = \frac{U_{n-p} + U_{n+p}}{2}$ | Every term is the quadratic mean of the terms which are équidistants from it. $U_n = \sqrt{U_{n-p}.U_{n+p}}$ |
| **Characteristic property # 2** | |
| Variation of **Linear** type | Variation of **Exponential** type |