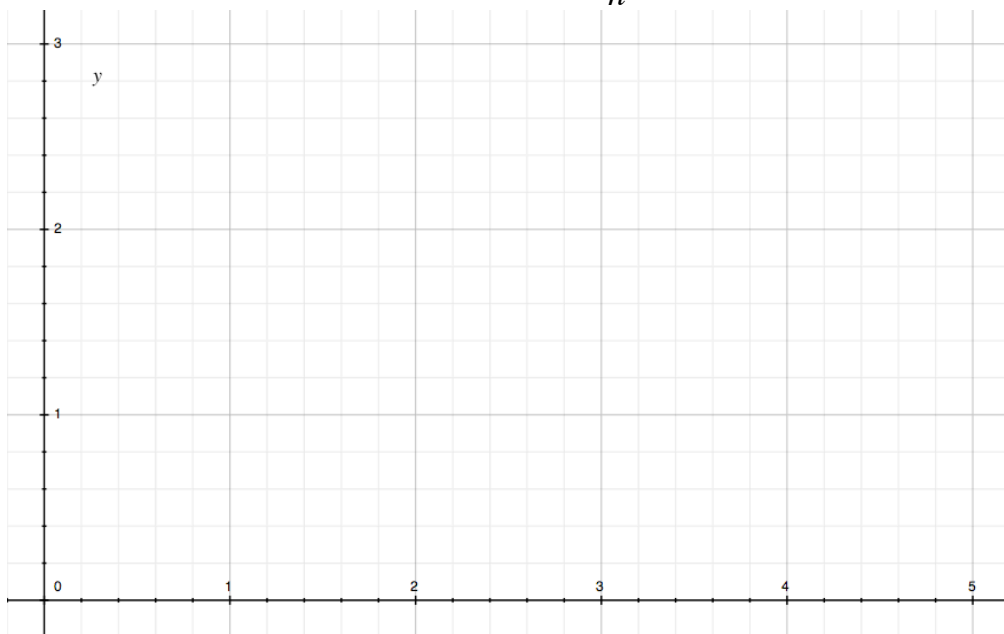


Numerical Sequences

Problem I : Let f be the function defined by $f(x) = \frac{2x+3}{x+4}$ for $x \geq 0$.

Study the Sequence defined by the formula $u_n = f(n) = \frac{2n+3}{n+4}$ for every $n \in \mathbb{N}$.

- a. Graph the function f on $[0 ; +\infty [$ and draw the first terms of the sequence (u_n) .
 Indicate from the graph whether or not the sequence is :
 - i. Monotonous (if yes how) :
 - ii. Bounded (if yes, what are the boundaries ?)
 - iii. Does-it seem to have a limit (if yes which one is it?)?
- b. Prove that (u_n) is increasing
- c. Prove that (u_n) is bounded by 0 and 2.
- d. Find for which value of n we have : $2 - \varepsilon < u_n < 2$ with $\varepsilon = 10^{-2}$
- e. Prove that for any $n \geq 1$ we have $|u_n - 2| \leq \frac{5}{n}$. Conclusion ?



Problem II : Let f be the function defined by $f(x) = \frac{2x+3}{x+4}$ for $x \geq 0$.

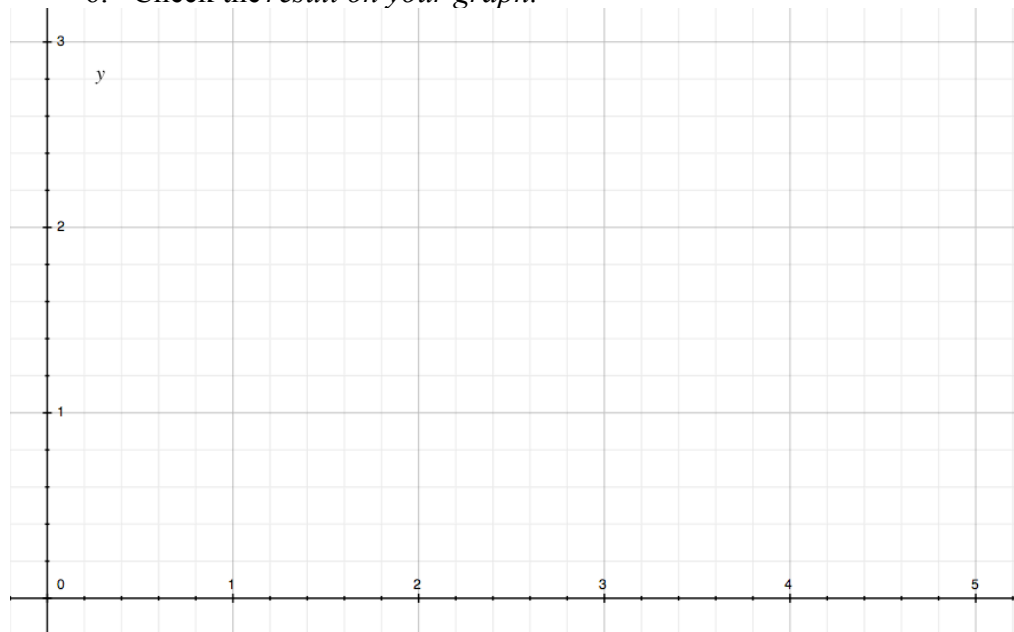
Study of the sequence (v_n) defined by $v_{n+1} = f(v_n) = \frac{2v_n+3}{v_n+4}$; $n \geq 1$ and $v_0 = 4$.

1. Graph the function f on $[0 ; +\infty [$ and draw the first terms of the sequence (u_n) .
 Find the coordinates of the intersection of (C_f) with the first bisector ($y=x$)
 Indicate from the graph whether or not the sequence is :
 - i. Monotonous (if yes how) :
 - ii. Bounded (if yes, what are the boundaries ?)
 - iii. Does-it seem to have a limit (if yes which one is it)?

2. Let $w_n = \frac{v_n - 1}{v_n + 3}$ for any $n > 0$.

Show that the new sequence (w_n) is a geometric sequence :

1. Find its first term and its reason.
2. Find the expression of w_n directly in function of n .
3. Deduct the limit of w_n .
4. Find the expression of v_n in function of w_n
5. Find the limit of v_n
6. Check the result on your graph.



3.