

I. The car and the bee :

The distance between two cities A and B is 120 Km.

A "jet-bee" leaves the city A to go towards B at an average speed of 80 km/h.

A car leaves the city B at an average speed of 40 Km/h, to go towards A.

1.) Find at what distance of A will the bee crash against the window of the car ?

2.) What time will it take before it crashes ?

Explain clearly your answer and draw the graph of the two movements on the chart.

When the Bee crashes it has flown twice as much as the car in the same time, because the Bee is flying at a speed which is double of that of the car.

So that if D is the distance from A to the crash we have : $D + D/2 = 120$ km,

So that $D = 80$ Km. Therefore the time is 1 hour from the departure.

3.) We now suppose that every time the bee hits the car window, it goes back to A and starts again towards B. Then it hits the car window again and goes back and forth indefinitely ... until the car reaches A.

What is the distance run by the fly until the car reaches A ?

Explain clearly your answer and draw the graph of the two movements on the chart.

First solution (no calculations !)

Because the Bee is flying twice as fast as the car, when the car will have travelled 120 Km, the Bee will have travelled the double : 240 Km.

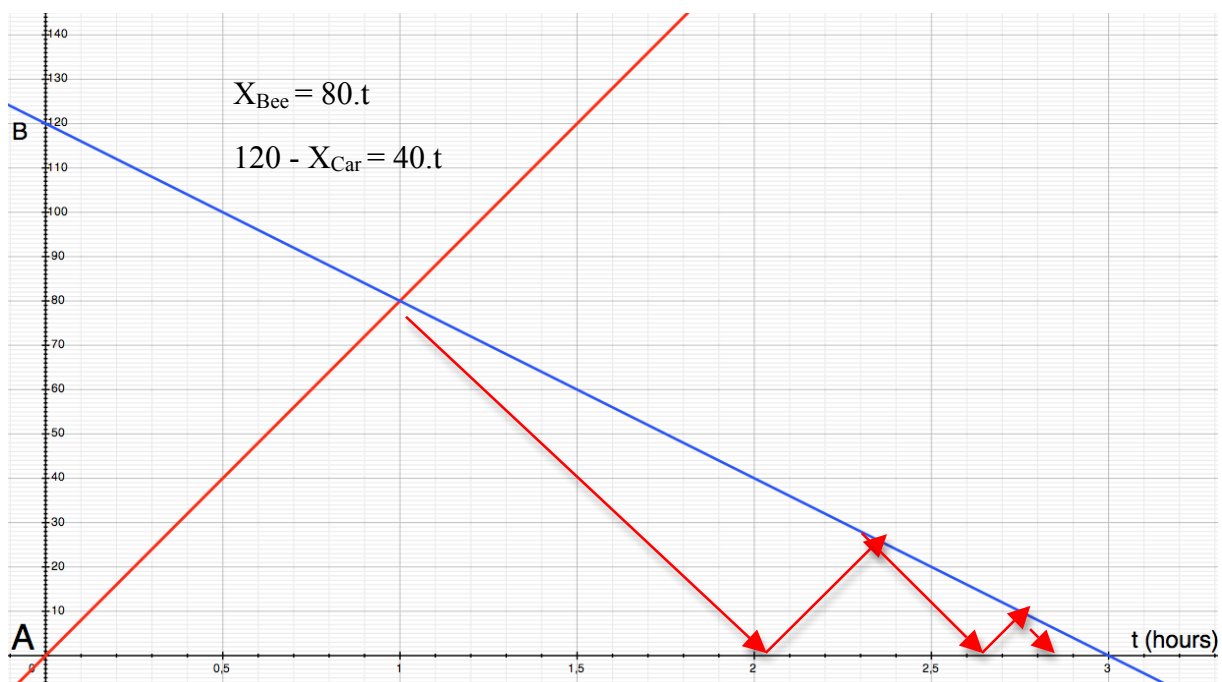
Second Solution : by observing the graph of the movements of the Bee, we can see that the distance run by the Bee is each time twice the ordinate of the point of impact on the car.

By geometrical observations we can say that each impact has an ordinate that is 1/3 of the previous impact ordinate.

So that the total distance is $D = 2[d_1 + d_2 + d_3 + \dots + d_n + \dots]$ with for any n, $d_{n+1} = d_n/3$

Hence :

$$D = 2d_1 \left[1 + \frac{1}{3} + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \dots + \left(\frac{1}{3}\right)^n + \dots \right] = 2 \times 80 \times \frac{1}{1 - \frac{1}{3}} = 240$$



III. Problem of economics optimization of fitness / Linear Programming.

Suzy wants to buy two kinds of sweets A and B.

The A kind costs 4 Yuans per Kg, the B kind costs 1 Yuan per Kg.

But she cannot spend more than 2 Yuans, and, because she is on diet, she cannot eat more than 1Kg per day.

Let x be the number of Kg for A and y the number of kilograms for B.

The A type produces 0,6 Kcal / Kg, and the B type 0,2 Kcal/Kg.

Question : What is the maximum Kcal that she would consume in these conditions ?

1. Explain (back page) why the constraints are represented by the following system:

$$\begin{cases} x \geq 0; y \geq 0 \\ 4x + y \leq 2 \\ x + y \leq 1 \end{cases} \quad \text{Conditions imposed by the maximum price of 2 Yuans and by the}$$

maximum weight of consumption per day.

and Total Kcal : $T = 0,6x + 0,2y$ (Kcal)

2. Graph the above inequalities below, and explain (back page) why the maximum Number of Kcal would be obtained for the values of x and y corresponding to the **vertex** of the domain of the allowed consumption

The domain allowed for the quantities $(x ; y)$ has a vertex in the point $(0,33 ; 0,66)$

Therefore when the quantity T of Kcal corresponding varies, it must stay inside the shade area and therefore the corresponding line cannot go above the point corresponding to the vertex of the allowed domain.

