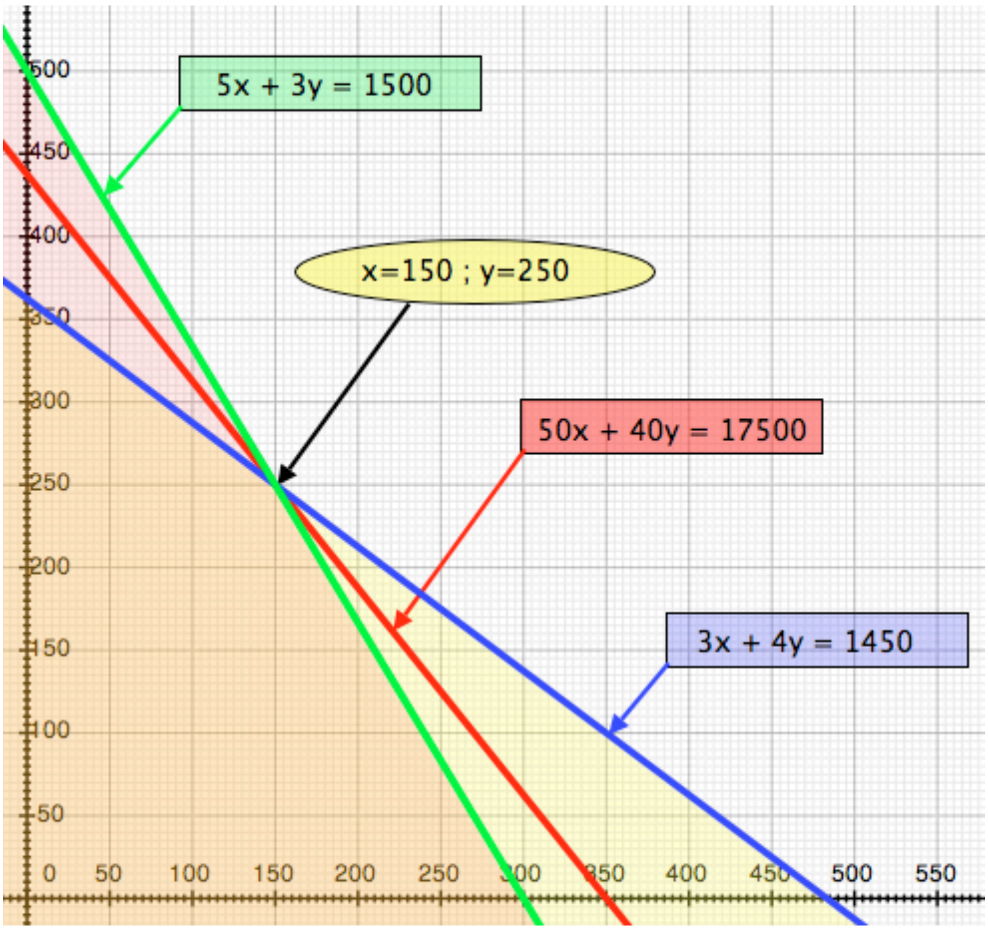
I – Linear Programming : *[30 pts] A publishing company is producing two kinds of software A and B on DVDs. It uses two machines for the production : one for the disk burning and one for the packaging.*
Let x be the number of DVDs of A type and y be the number of DVDs of B type.
The burning machine takes 3 minutes to burn the DVD A and 4 minutes for B, but can work only for 24 hours and 10 minutes per series.
The packaging machine takes 5 minutes for the DVD A and 3 minutes for B, but it can work only for 25 hours in a row. Each DVD A is sold 50 Yuans and each DVD B is sold 40 Yuans.

1. *Write the system of inequalities corresponding to this production.*
2. *Draw the lines corresponding to the production of each machine.*
3. *Shade the area corresponding to these conditions of production.*
4. *Write the equation corresponding to the total amount sold for this production.*
5. *Find the maximum number of DVD A and B which can be produced.*
6. *Draw the line of the sales corresponding to that maximum production.*

- *System of inequalities corresponding to the conditions of this production :* $\begin{cases} 3x + 4y \leq 1450 \\ 5x + 3y \leq 1500 \\ x \geq 0 \,;\, y \geq 0 \end{cases}$
- *Total amount of money engaged for this production of x DVD/A and y DVD/B*
  $$S = 50x + 40y$$
- *S is maximum for the point corresponding to the vertex of the authorized domain.*
  x = 150 DVD/A & y = 250 DVD/B => S = 17500 (Yuans) per series.



II– Parabolas and Hyperbolas : [40 pts] $f(x)=-\frac{1}{4}x^2-x+3$ $g(x)=\frac{2x+12}{x+4}$

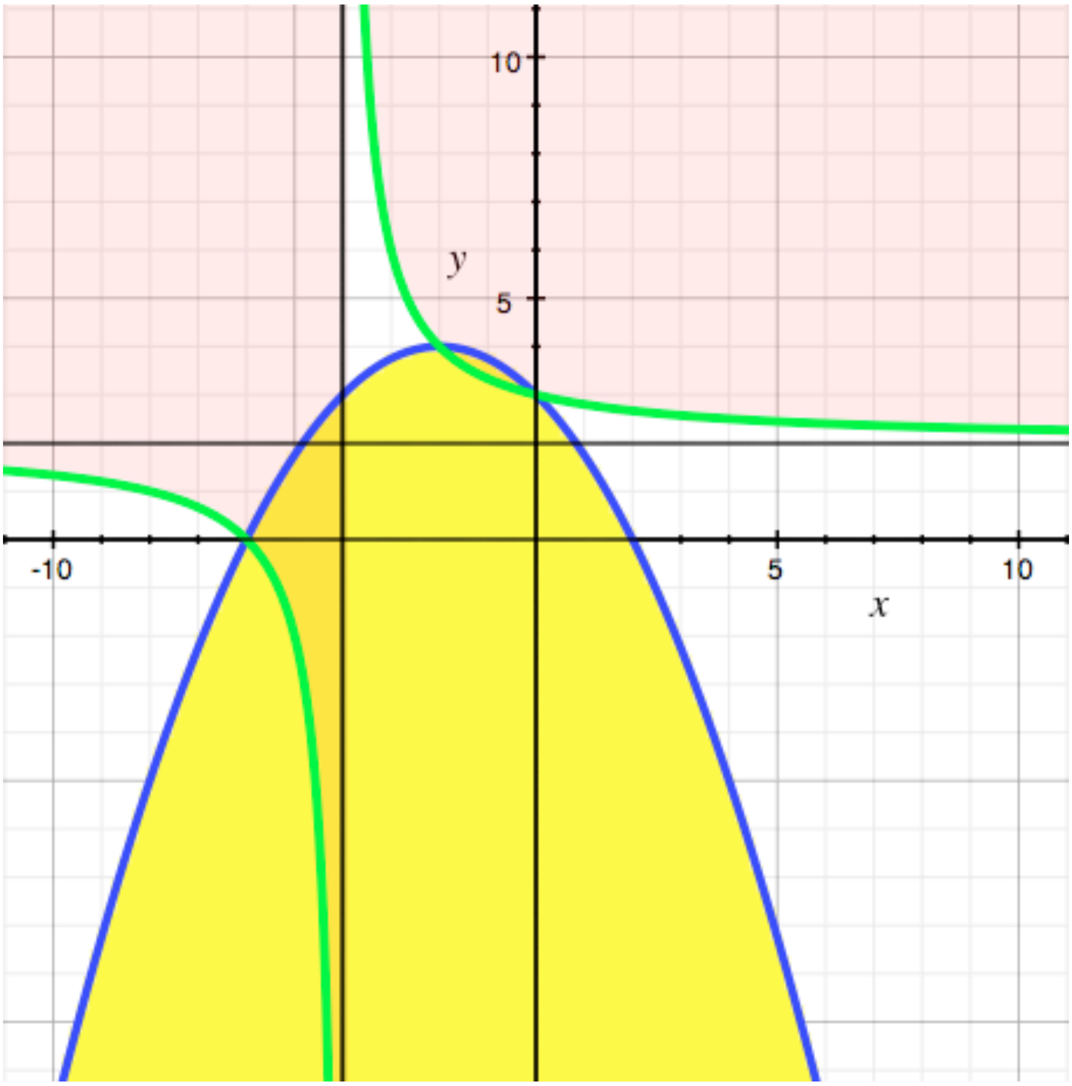
1. *Draw carefully the graphs of the two functions in the same system of coordinates. Show the axis of symmetry of the Parabola and the asymptotes of the Hyperbola*
2. *Calculate and show the coordinates of intersections with the 0x and the 0y axes.*
3. *Solve the equation f(x) = g(x) to find the coordinates of the intersection points of the Parabola and the Hyperbola.*
4. *Shade the area of points (x;y) corresponding to the system of inequalities : $y \le f(x)$ & $y \ge g(x)$*

- *Axis of Symmetry of the Parabola : x= -2*
- *Center of Symmetry of the Hyperbola : (-4 ; 2)*
- *Intersection of the Parabola with the x axis : y = 0 $\Leftrightarrow$ x = -6 or x = 2*
- *Intersection of the Parabola with the y axis : x = 0 $\Leftrightarrow$ y = 3*
- *Intersection of the Hyperbola with the x axis : y = 0 $\Leftrightarrow$ x = -6*
- *Intersection of the Hyperbola with the y axis : x = 0 $\Leftrightarrow$ y = 3*
- *Intersection of the two curves :*

$$-\frac{1}{4}x^2 - x + 3 = \frac{2x+12}{x+4} \Leftrightarrow -\frac{1}{4}(x+6)(x-2)(x+4) = 2(x+6)$$

$$\Leftrightarrow (x+6)[-\frac{1}{4}(x-2)(x+4)-2] = 0 \Leftrightarrow (x+6)[-\frac{1}{4}x^2 - \frac{1}{2}x] = 0 \Leftrightarrow x(x+6)(x+2) = 0$$

$$\Leftrightarrow x = 0\ (y=3) \quad or \quad x = -2\ (y=4) \quad or \quad x = -6\ (y=0)$$



III – Associated Functions and Transformations : *graph the following functions in the system below*

$$f1(x)=\tfrac{1}{4}x^2-|x|-3 \quad ; \quad f2(x)=-\sqrt{(x-3)^2}+4; \quad f3(x)=-\sqrt{9+9x}+6$$

- *$f_1(x) = f(|x|)$ with* $f(x)=\frac{1}{4}x^2-x-3$, *because* $(|x|)^2 = x^2$.
  Therefore the graph of f_1 is associated to a parabola

- $f_2(x)=-\sqrt{(x-3)^2}+4=-|x-3|+4$. *Therefore the graph of f_2 is associated to the opposite of the absolute value function, translated from the origin by the vector V(3;4).*

- $f_3(x)=-\sqrt{9+9x}+6=-3\sqrt{x-(-1)}+6$ *Therefore the graph of f_3 is associated to the function defined by the opposite of 3√x and then translated from the origin by the vector V(-1;+6).*