Elementary functions and Numerical Sequences

Problem I – Around an island in the Southern Pacific Ocean, there are some seaweeds and some coral which develop together, but while the amount of seaweeds increases, the amount of corals is decreasing.

Eventually the corals will die away when there are too many seaweeds around. Scientists have studied the evolution of both populations of these living creatures. They found that the area covered by the seaweeds is **increasing by 15%** every year, and that the area covered by the coral is **decreasing of 15000 m^2** every year.

On January 1st of 2000 the area covered by the seaweeds was **150,000 m^2** and the area covered by the coral was of **350,000 m^2**.

Let U_n represent the area covered by the seaweeds in the year 2000 + n. $U_0 = 150,000$.

Let V_n represent the area covered by the coral in the year 2000 + n. $V_0 = 350,000$.

1. **Study of the sequence (Un)** :
   a. Calculate U_1 and U_2 — 2pts

   b. Write the relationship between U_{n+1} and Un. — 2pts

   c. What is the nature of the sequence (U_n) ? — 2pts

   d. Write the value of U_n directly in function of n. — 2pts

   e. What is the area covered by the seaweeds in 2010 ? (*Use your pocket calculator*) — 2pts

2. **Study of the sequence (V_n) :**
   a. Calculate V_1 and V_2 — 2pts

   b. Write the relationship between V_{n+1} and V_n. — 2pts

   c. What is the nature of the sequence (V_n) ? — 2pts

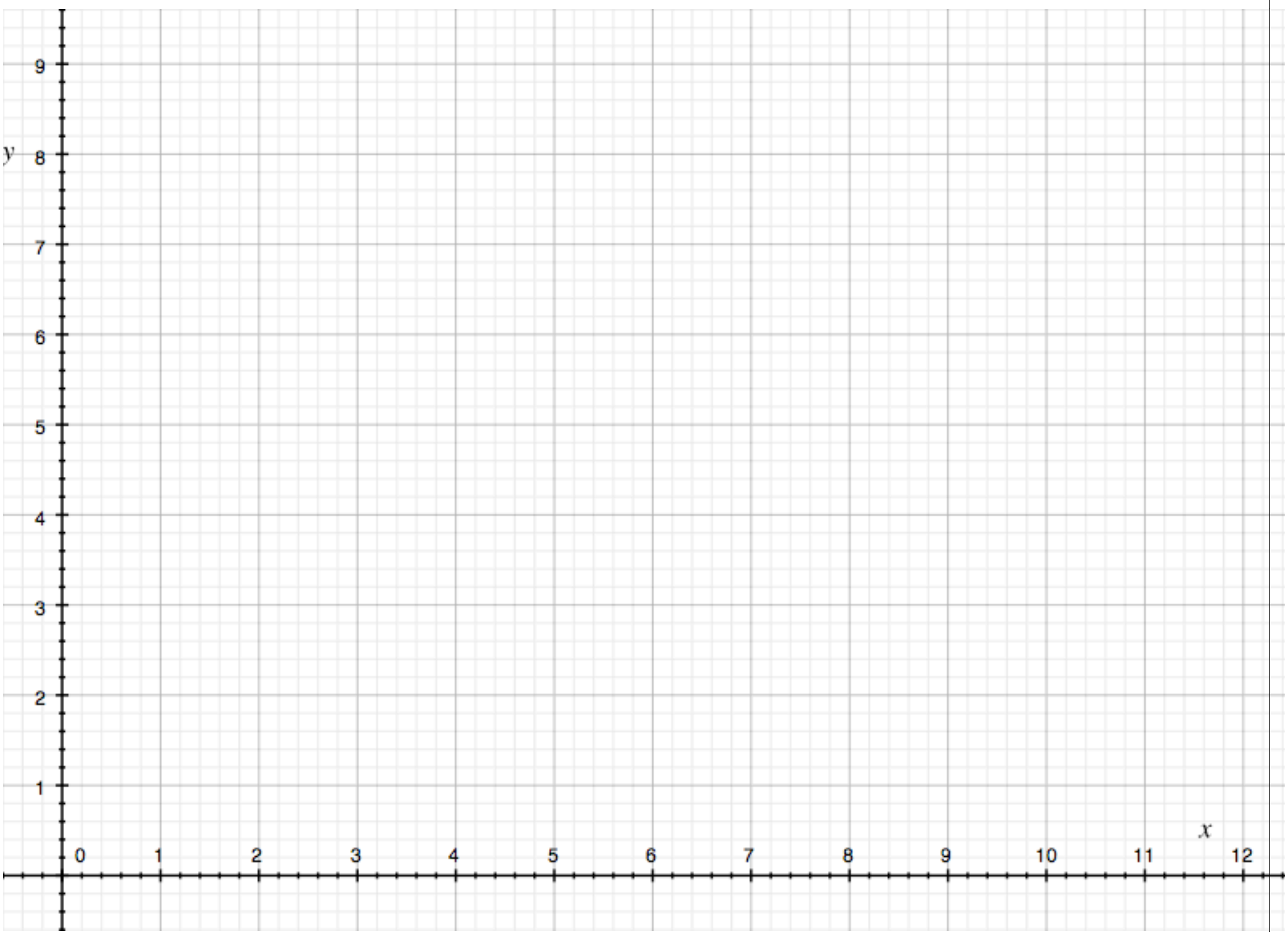
   d. Write the value of V_n directly in function of n. — 2pts

   e. What is the area covered by the coral in 2010 ? — 2pts

Problem II – Some scientists have described the evolution of a kind of small desert mice by the following sequence : U_n represents the number of thousands of mice at the year 2000+n. To survive the mice have to hide against the desert snake wich eat them up but they don't allways escape …

$$U_0 = 10 \text{ and } U_{n+1} = f(U_n) \text{ with } f(x) = \frac{1}{4}x(12-x) \text{ for } 0 \le x \le 12 \quad \therefore \quad U_{n+1} = \frac{1}{4}U_n(12-U_n)$$

1.	*Draw **carefully** the graph of the function f.*	6pts
2.	***Calculate** U_1 , U_2, U_3 .*	2pts
3.	*Show the **construction** of the first tems U_1 , U_2, U_3 U_4 , U_5, U_6 …. on the graph*	6pts
4.	*Is the sequence (U_n) **monotonous** ? (If yes say how).*	2pts
5.	*Is the sequence (U_n) **bounded** if yes give the boundaries)*	2pts
6.	*Is the sequence (U_n) converging towards a finite **limit** (if yes, give that limit).*	2pts

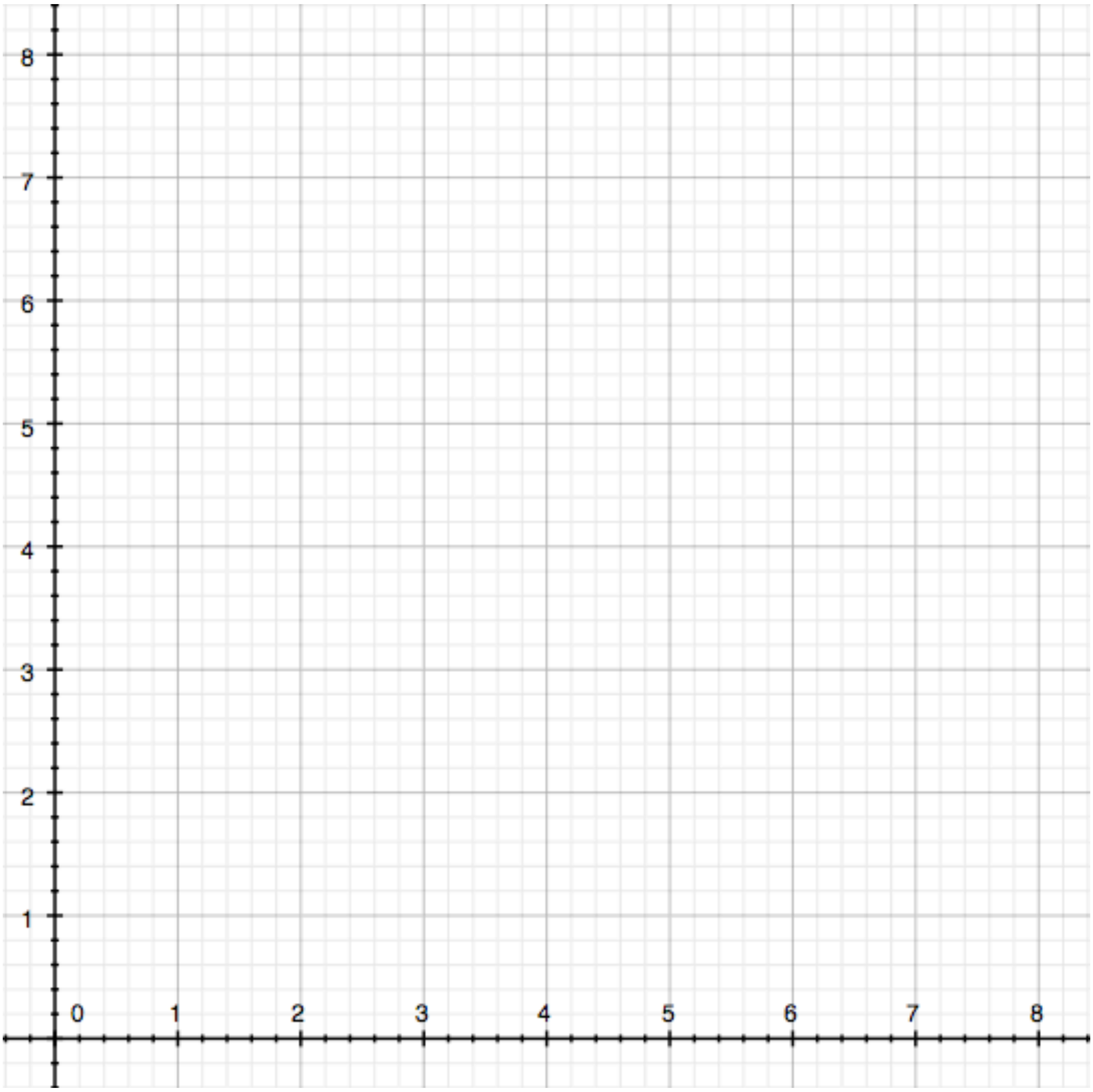


All calculations must shown below or on back of this page

Problem III : Let f be the function defined by $f(x) = \dfrac{x+8}{2x+1}$ for $x \geq 0$.

Study of the sequence (v_n) defined by $u_{n+1} = f(u_n) = \dfrac{u_n+8}{2u_n+1}$; $n \geq 0$ and $u_0 = 7$.

1.	Graph the function f on [0 ; 8] ***(next page)***	8pts
2.	Show the construction of the **first terms** of the sequence (u_n) ***(next page)***	8pts
3.	Find the coordinates of the **intersection** (Cf) with the first bisector (y=x)	3pts
4.	Indicate from the graph whether or not the sequence is : i. **Monotonous** (if yes how) :	2pts
	ii. **Bounded** (*if yes, what are the boundaries ?)*	2pts
	iii. **Does-it seem to have a limit (if yes which one is it?)?**	2pts
5.	Let $v_n = \dfrac{u_n - 2}{u_n + 2}$ *for any n>0.* Prove that the new sequence (v_n) is **geometric** :	5pts
	i. Give its **first term** v_0 and its **reason** q	2pts
	ii. Give the **expression** of v_n directly in function of n.	2pts
	iii. Deduct the **limit** of v_n.	2pts
	iv. Find the **expression** of u_n in function of v_n	2pts
	v. Find the **limit** of u_n	2pts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