Definition and Construction of an Equilateral Hyperbola (Part 1)

Let f be the function defined by : $f : x \mapsto y = \dfrac{A}{x}$

I- **Algebraic properties**:

1°) **Odd** function: for any $x \in \mathbb{R}^*$, $f(-x) = - f(x)$.

2°) Rate of growth **non constant** : $T_{[f,(x_1,x_2)]} = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{-A}{x_1 x_2}$

3°) Sign of T = Signe of *(- A)* on $[0 ; +\infty[$ and on $]- \infty ; 0]$

4°) Variation Chart :

A > 0						A < 0				
x	$-\infty$	-1	0	1	$+\infty$	x	$-\infty$	-1	0	1	$+\infty$
T		$-$		$-$		T		$+$		$+$	
f	$0^{(-)}$ ↘ $-A$ ↘ $-\infty$			$+\infty$ ↘ A ↘ $0^{(+)}$		f	$0^{(+)}$ ↗ $-A$ ↗ $+\infty$			A ↗ $0^{(-)}$	↘ $-\infty$

II- **Geometric Properties** :

1°) The curve representing f is symmetrical through the Origin of axes. This curve is called an **Equilateral Hyperbola** because of its central Symmetry and because x and y vary in reverse directions. (The word hyperbolic means something exaggerated)

2°) The Hyperbola cuts the 1st bisector $(y = x)$ in I $(\sqrt{A};\sqrt{A})$ *if* $A > 0$ or $(\sqrt{-A};-\sqrt{-A})$ *if* $A < 0$

3°) On I the Hyperbola is tangent to the line perpendicular to the bisector.

4°) The Hyperbola contains the point J(1 ;A) and its symmetrical point (-1 ;-A) through O

5°) If $A > 0$ the 1st bisector $(y = x)$ is an axis of symmetry.
   If $A <$ the 2nd bisector $(y = - x)$ is an axis of symmetry.

6°) When |A| is very large compared to 1 (|A| >> 1), The Hyperbola is very wide and away from O. Inversely if |A| <<1 the Hyperbola is very narrow and close to 0.

7°) The Hyperbola contains absolutely no segment of a straight line.

8°) The Hyperbola has two *asymptotes* which are the axes of coordinates (Ox) et (Oy).

$y = \dfrac{1}{x}$

$y = -\dfrac{1}{x}$
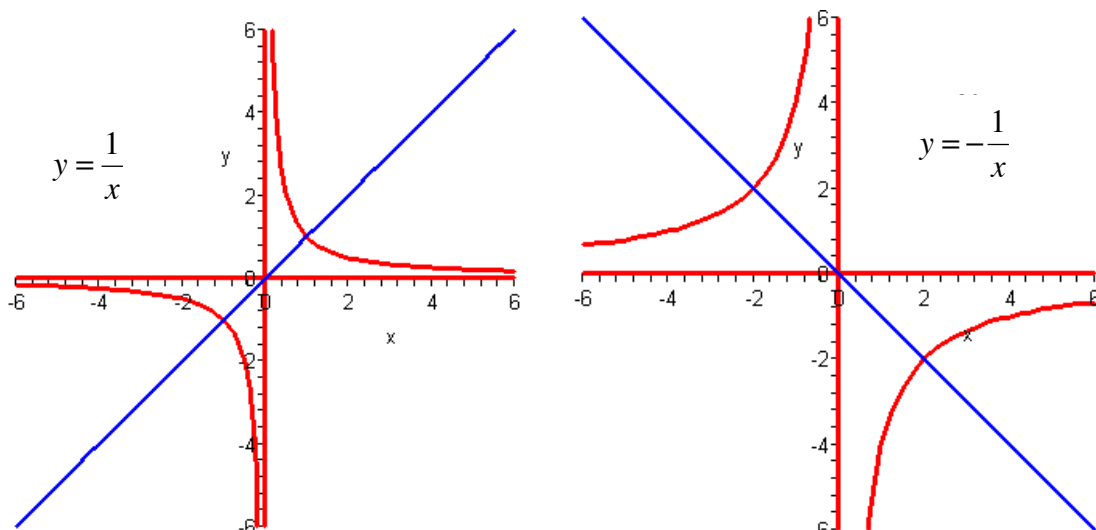
Hyperbolas & Homographic Functions (Part 2)

Homographic functions are those defined by the type : $\boxed{f : x \mapsto y = \dfrac{ax+b}{cx+d}}$ with $c \neq 0$

That expression can take one or the other of the following forms :

$(H_1)\quad y = \dfrac{A}{x}$

$(H_2)\quad y = \dfrac{A}{x} + H$

$(H_3)\quad y = \dfrac{A}{x-L}$

$(H_4)\quad y = \dfrac{A}{x-L} + H$

$(H_5)\quad y = \dfrac{ax+b}{cx+d}$

1°) Transformation from (H_1) to (H_2) is a **Translation** defined by the vertical vector $H\vec{j}$
(parallel to the (Oy) axis. (H_2) intercepts (Oy) in y = H. *(H = « Hight » ; L = « Length »)*

2°) Transformation from (H_1) to (H_3) is a **Translation** defined by the horizontal vector $L.\vec{i}$
(parallel to the (Ox) axis).

3°) Transformation from (H_1) to (H_4) is a **Translation** of vector $\vec{V} = L.\vec{i} + H.\vec{j}$

4°) The Hyperbola (H_4) has its center of symmetry in O'(L ;H).
Let $X = x - L$ and $Y = y - H$ then $Y = A/X$ which means that (H_4) is Symmetrical through O'
Therefore (H_4) is drawn in the system (O'X ; O'Y) just like (H_1) in the system (Ox,Oy).

5°) To build the Hyperbola (H_5) one can choose between two methods :

    a. Change (H_5) into (H_4) by breaking the fractions in simple elements (cf. examples).

    b. Find the coordinates of the center with the formulas $\boxed{O'(L = \dfrac{-d}{c} ; H = \dfrac{a}{c})}$

        then find the intersections with the two axes : (Oy) : $(0 ; \dfrac{b}{d})$ and (Ox) $(\dfrac{-b}{a} ; 0)$.

$$y = \frac{x-2}{x+2} = \frac{-4}{x+2} + 1$$