The Golden Number in Geometry

Problem I : in a rectangle of sides x (length) and y (width) we want to divide it such that the square inscribed in the rectangle determines a new rectangle of which the sides are in the same ratio.

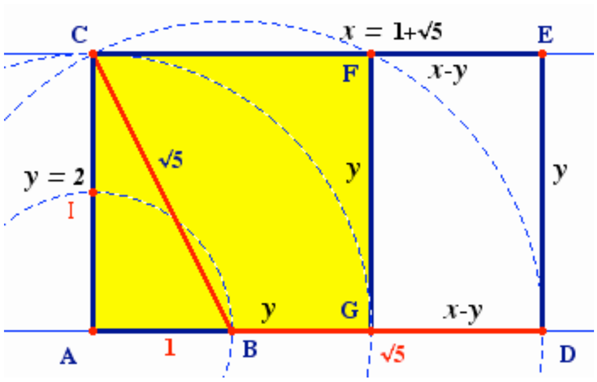
1) Find what should be the value of the ratio $r=\frac{x}{y}$ such that : $r=\frac{x}{y}=\frac{y}{x-y}$

Answer : $r=\frac{x}{y}=\frac{y}{x-y}=\frac{1}{\frac{x}{y}-1}=\frac{1}{r-1}\Leftrightarrow r=\frac{1}{r-1}\Leftrightarrow r^2-r-1=0\ (r>0)\Leftrightarrow r=\frac{1+\sqrt{5}}{2}$

This number is called the **Golden Number**. *It's Inverse* $\frac{1}{r}=\frac{2}{1+\sqrt{5}}=\frac{\sqrt{5}-1}{2}$ *is called the* ***Golden section.***

2) **Construction of the Golden Rectangle** :

1. Rectangle Triangle ABC is such that AC = 2 AB. Therefore BC = √5.
2. From B draw a circle of radius = BC which cuts AB in D. Therefore AD = 1+√5
3. Complete the picture by drawing the rectangle ADEC and the square AGFC.
4. We check that the ratio $\frac{AD}{AC}=\frac{1+\sqrt{5}}{2}=r$

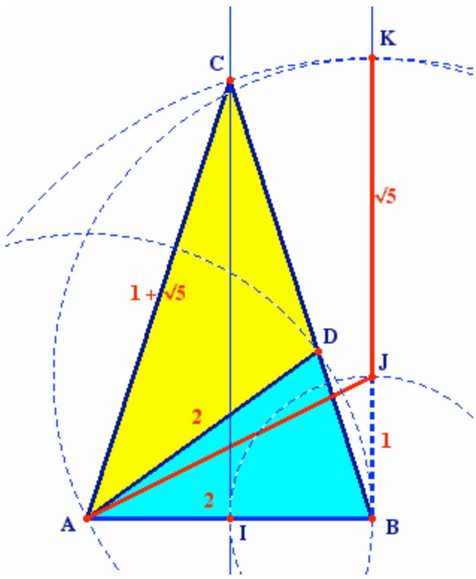


Problem II : in an isosceles triangle of sides x (length) and y (width) we want to divide it such that the isosceles triangle inscribed in the triangle determines a new isosceles triangle of which the sides are in the same ratio.

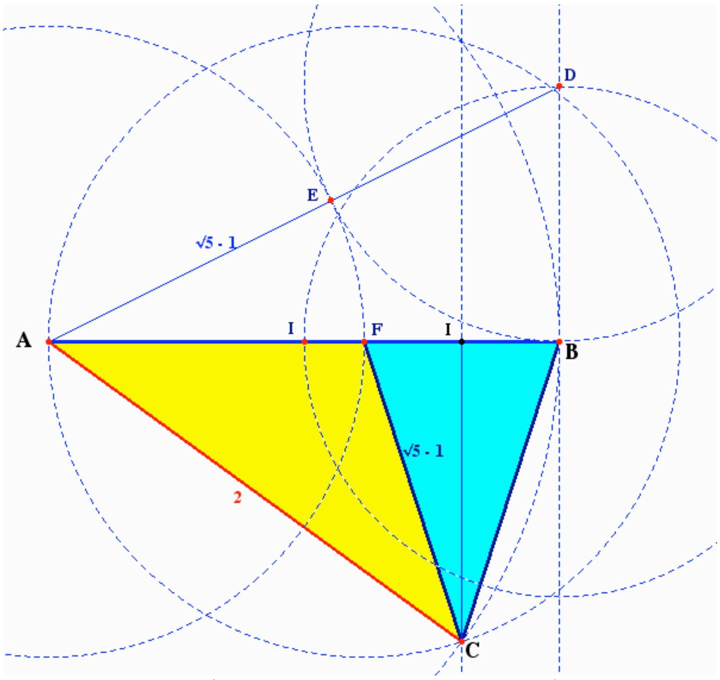
Construction of the golden triangle *(uses the same simple technique as above)*

1. Rectangle Triangle ABJ is such that AB = 2 BJ. Therefore AJ = √5.
2. From J draw a circle of radius = AJ which cuts BJ in K. Therefore BK = 1+√5
3. From B draw a circle of radius =BK
4. Draw the perpendicular bisector of AB, which cuts the previous circle In C.
5. Complete the picture by connecting BC and AC.
6. The circle of center A and radius AB cuts BC in D, such that the triangle ABD isosceles in A.
7. Check that the sides are in the golden ratio :

$$\frac{AC}{AB}=\frac{1+\sqrt{5}}{2}=r \text{ and } \frac{AD}{BD}=\frac{2}{\sqrt{5}-1}=r$$



Other solution for the construction of the Golden Triangle
(many students have presented this solution but with bad confusions and very often their construction was incomplete and/or NOT JUSTIFIED



In this presentation the construction of the golden section AF from the rectangle triangle ABD with AB = 2 BD gives AF= AE = AD-DE =√5-1. (F is the golden point on AB). That's OK.
But then many students build the circle centered in F of radius = √5 -1 and the circle centered in A of radius equal to AB to find the point C.
It is correct, but they did not prove that BC = CF = √5 – 1 which is necessary.

Others students used the intersection of the perpendicular bisector of BF with the circle centered in F of radius = AF but they did not prove that AC =2.

The best solution was to build the intersection of the circle centered in A of radius AB = 2 and the circle centered in B of radius equal to FA.

Note : Some students have studied the special picture of the golden triangle and proved that the vertex of any isosceles golden triangle is equal to 36° (the base angles are 72°). This is right and the golden ratio gives us a simple way to calculate the exact value or the sine and cosine of 18°. In the picture page 1 we see that :

$$\widehat{ACI} = \frac{1}{2}\widehat{ACB} = 18° \Rightarrow \sin 18° = \frac{AI}{AC} = \frac{1}{1+\sqrt{5}} = \frac{\sqrt{5}-1}{4}$$

$$and \quad \cos 18° = \sqrt{1-(sin18°)^2} = \sqrt{1-\frac{6-2\sqrt{5}}{16}} = \frac{\sqrt{10+2\sqrt{5}}}{4}$$

$$\widehat{BAC} = 90-18 = 72° \Rightarrow \cos 72° = \sin 18° = \frac{\sqrt{5}-1}{4}$$