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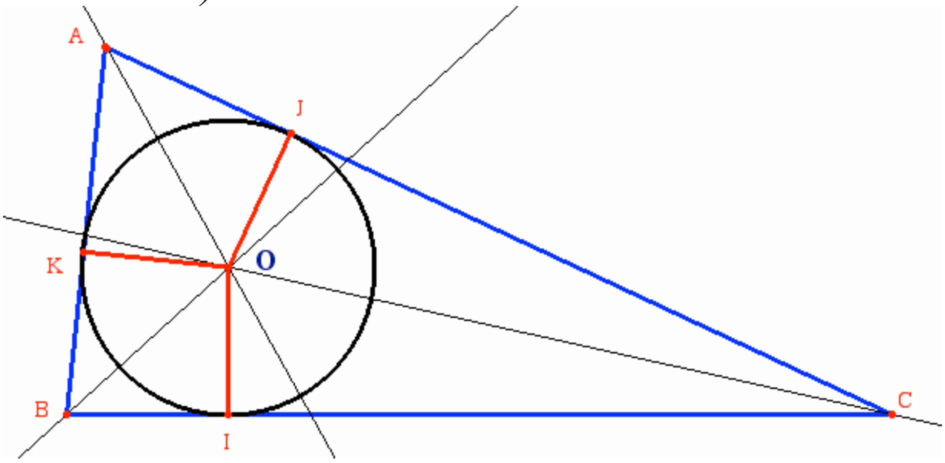
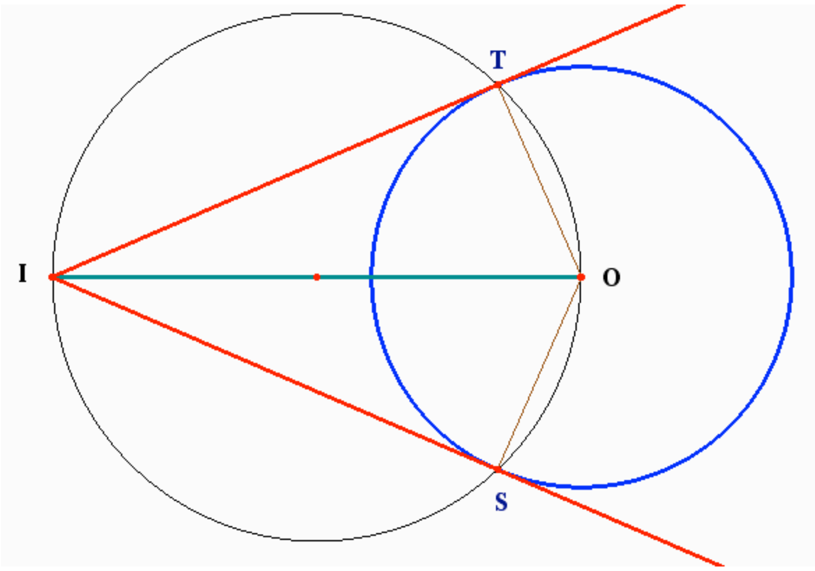
Problem 1 : use a compass and a ruler to carefully build the circle inscribed in this triangle *(show the construction lines and explain your construction).*



Solution : *by definition any point of the bisector of an angle is situated at an equal distance of the sides of the angle. Therefore the point O intersection of the three bisectors is at an equal distance from the sides of the triangle : OI = OJ = OK then the circle centered in O and of radius R=OI is tangent to the three sides of the triangle. This cirle is called the* ***inscribed circle*** *of ABC.*

Problem 2 : given the circle (C), and a point I outside (C), use a compass and a ruler to carefully construct the two tangent lines to the circle from I.
[*find the points K and K' on (C) so that the lines (IK) and (IK') be tangent to (C)*]



Solution : the contact points T and S with (C) are at the intersection of the circle of diameter OI with (C), because the triangles OIT and OIS must be rectangular triangles in T and S.

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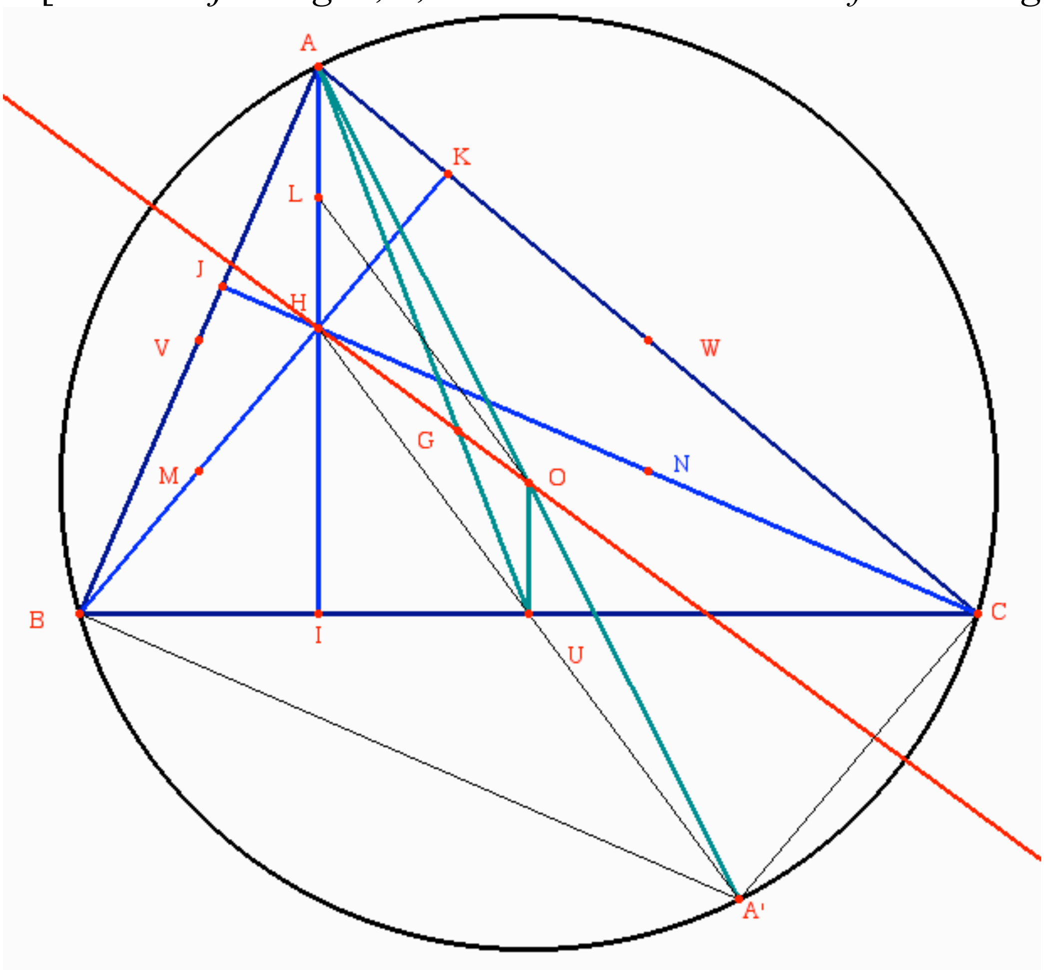
Problem 3 : given the triangle ABC, let H be the interception of its Heights, and O be the center of the circumscribed circle, U the middle of BC, and G the intersection of OH and AU.

1°) Prove carefully *(on back of the page)* that GH = 2 GO.

2°) Show why G is the center of gravity of the triangle.

[*The line joining O,G,H is called* Euler's *line of the triangle*]



Solution : *we already know that HBA'C is a parallelogram. Therefore U, middle of [BC] is also the middle of HA'.*
Hence [OU] is the line joining the middle point of the sides of the triangle AHA', which means that OU is parallel to AH and OU = ½ AH.
Then the triangles GOU and GHA are similar triangles (two equal angles) with ratio ½. Hence GH=2GO and GA=2GU which proves that G is the center of gravity of the triangle ABC (well known characteristic property of G).

<u>Conclusion</u> : *in any triangle ABC, the orthocenter, the center of gravity and the center of the circumscribed circle are always on the same line, called "***Euler's line***" of the triangle.*

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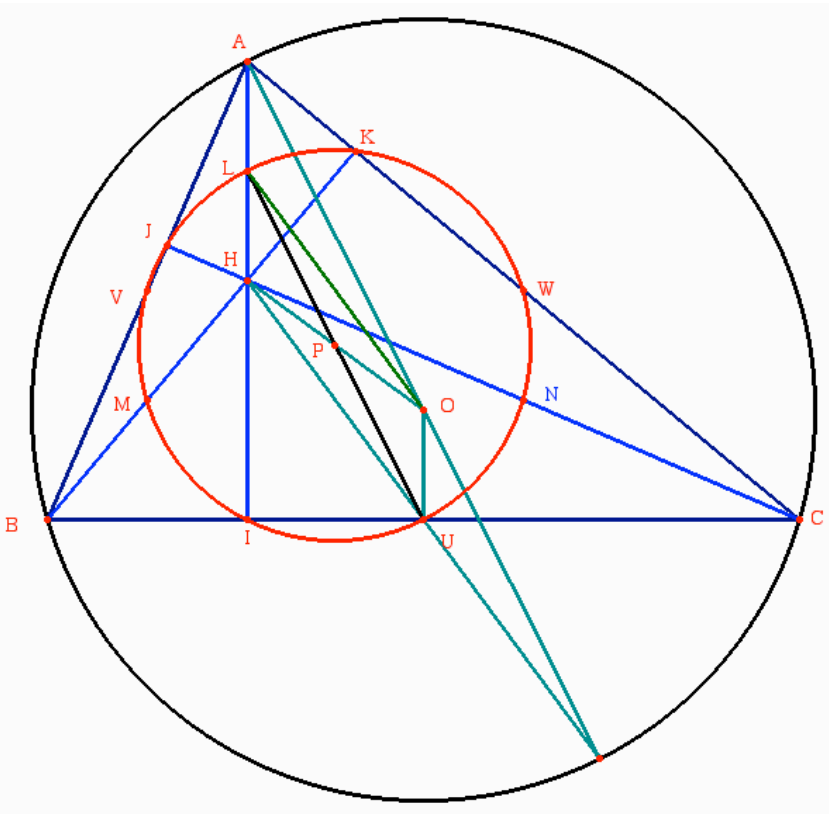
Mathematics - Geometry ++ Junior 8

Problem 4 : given the triangle ABC, let H be the intercection of its Heights, and O be the center of the circumscribed circle, L the middle of AH, and P be the middle of OH.

1°) Prove carefully *(on back of the page)* that PL = PU = PI.

2°) By the same method prove that the circle centered in P and of r = R/2 where R is the radius of (C), contains the 9 points : I,J,K,U,V,W,L,M,N *(this circle is named "*Euler circle*" of the triangle)*



Solution : *The quadrilateral (OUHL) is a parallelogram, because the two sides OU and LH are both parallel and equal. (see problem 3).*
Then the intersection of the diagonals of (OUHL) is the middle of [UL] therefore PU = PL ; and PL = PI because (UIL) is a right triangle in I.
In the parallelogram (OULA), one can see that LU = OA = R (radius of the circle circumscribed to ABC.
Hence the circle (E) centered in P (middle of OH) and of radius r = R/2 contains the 3 points U,I,L.
By the same reasoning applied to the other sides of the triangle we can say that (E) contains the other points K,W,M and J,V,N.

<u>*Conclusion*</u> *: in any triangle ABC, the circle centered in the middle of [OH] and of radius half or that of circumscribed cercle (C), contains the 9 points* I,J,K,U,V,W,L,M,N. *This circle is named* "**Euler's Circle**" *of the triangle.*