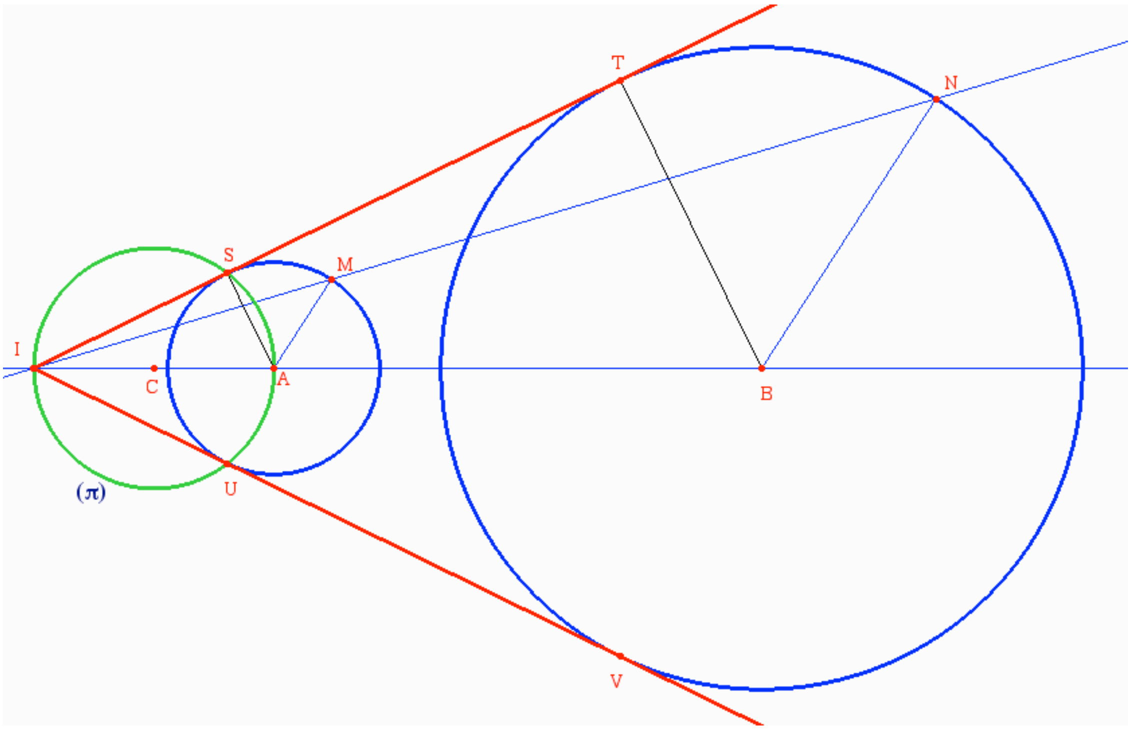
北京景山学校
jiguanglaoshi@gmail.com

Construction of the common tangents to 2 circles

Problem I : external common tangent : use the figure below to construct the points of external contact between the circle and their common external tangent line. ***Explain carefully your construction.***



Solution : *for any point M chosen on the first circle (C_1) and the corresponding point N on (C2) such that (BN) be parallel to (AM) in the same direction. The line (MN) cuts the line (AB) in one point I outside the two circles. Hence the triangles IAM and IBN are similar (two equal angles). This remains true whatever be M on (C_1), and especially when M is on the point S and N on T, that we are looking for.*

Then to build S and T we need to proceed as follow :

- *choose M on (C_1) and draw (BM) // to (AM).*
- *draw the line of the centers (AB) and mark I at the intersection of (AB) and (MN).*
- *Then we are back to the problem of drawing a tangent line to a circle from one outside point, then :*
  - *find the middle of [IA] and draw the circle (π) of diameter [IA] which cuts (C_1) in S and U.*
  - *draw the line (IS), which touches the circle (C_2) in T.*
- *(IS) is the first common tangent line to the two circles.*
- *(IU) is the second common tangent line to the two circles.*

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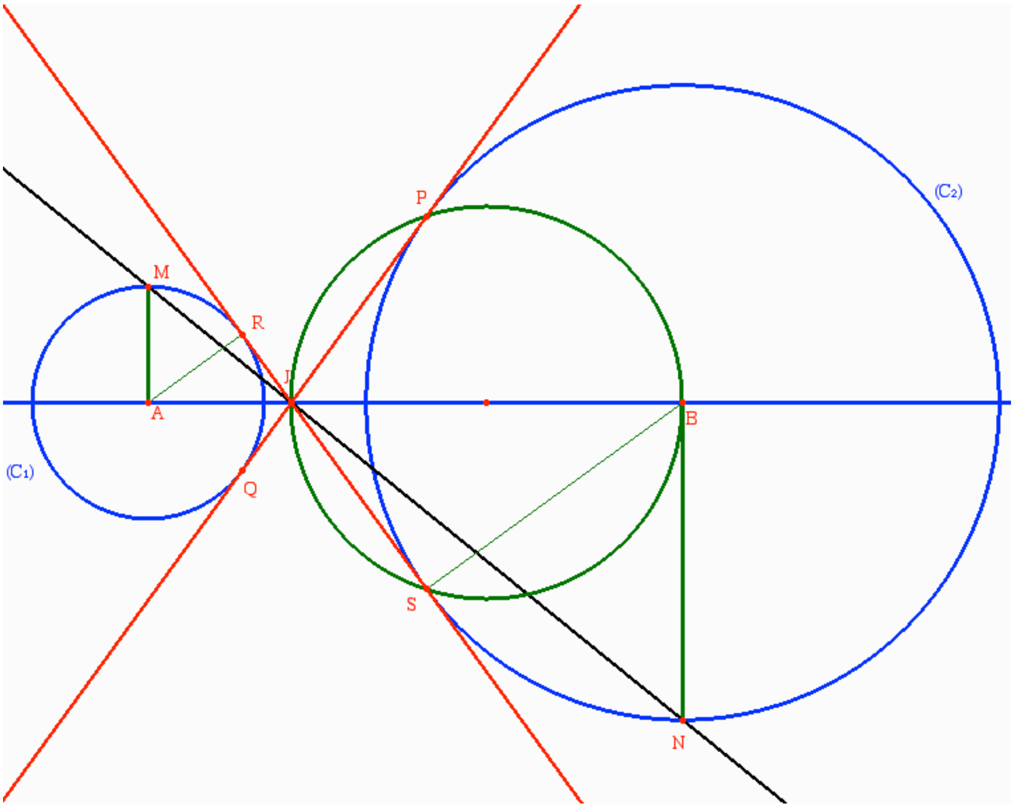
Problem II : Internal common tangent : use the figure below to construct the points of external contact between the circle and their common internal tangent line. ***Explain carefully your construction.***

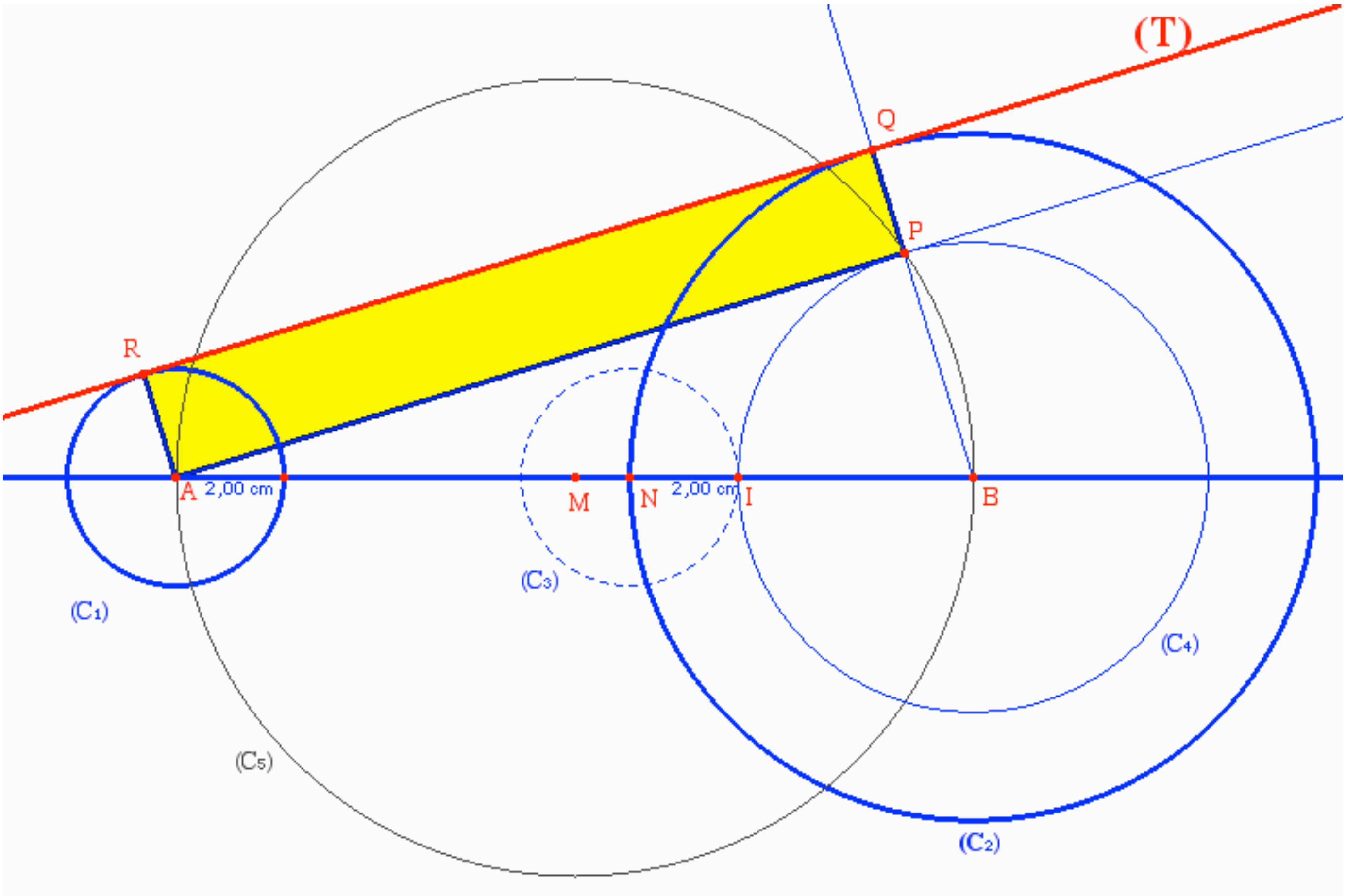


Solution : *similarly to the previous construction, let's pick a point M on (C_1) and the corresponding point N on (C_2) such that (BN) be parallel to (AM) but in the opposite direction. The line (MN) cuts the line (AB) in J, between A and B. Hence the triangles (JAM) and (JBN) are similar (two equal angles). This remains true whatever be M on (C_1), and especially when M is on the point S, and N comes on R, that we are looking for.*

Then to build S and R we need to proceed as follow :

- *choose M on (C_1) and draw (BM) // to (AM) in opposite directions.*
- *draw the line of the centers (AB) and mark J at the intersection of (AB) and (MN).*
- *Then we are back to the problem of drawing a tangent line to a circle from one outside point, then :*
    - *find the middle of [IB] and draw the circle of diameter [IA] which cuts (C_2) in S and P.*
    - *draw the line (IS), which touches the circle (C_1) in R.*
- *(IS) is the first common internal tangent line to the two circles.*
- *(IP) is the second common tangent line to the two circles in P and Q.*

Solution II for the construction of the common tangent.



1. *Draw the line of the centers (AB), mark the points M, middle of [AB], and N.*
2. *Draw a circle (C_3) centered in N of same radius R_1 than the circle (C_1).*
   It cuts [AB] in I.
3. *Draw the circle (C_4) centered in B and of radius $r = R_2 - R_1$.*
4. *Draw the circle (C_5) centered in M of diameter [AB], it cuts (C_4) in P.*
5. *Draw the line (AP), which is tangent to the circle (C_4).*
6. *Draw the line (BP) which is perpendicular to (AP). It cuts (C_2) in Q.*
7. *Draw the parallel line (T) to (AP) by Q.*
8. *Draw the perpendicular line to (T) by A. It cuts (T) in R.*

Hence the quadrilateral (APQR) is a rectangle (opposite sides are parallel and one right angle). Then we have [AR]=[PQ] = R_1. which means that R belongs to (C_1) and that (T) is also tangent to (C_1).