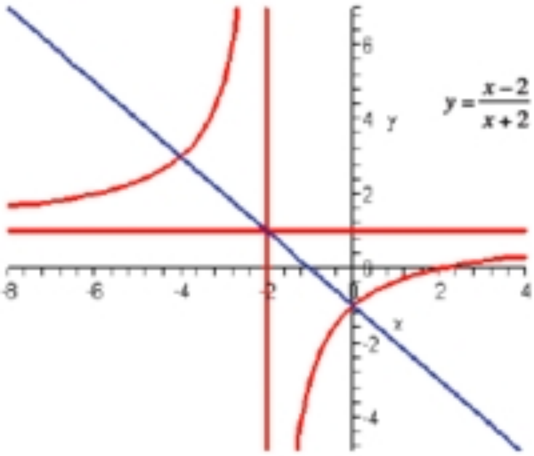
Asymptotes

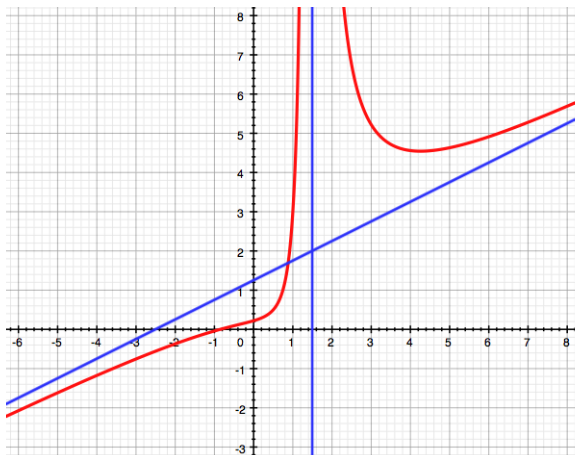
渐近线–漸近線

Definition : *a curve (Δ) is said to be an asymptote of curve (C) when the following is true: as one moves along (C) in some direction, the distance between it and the asymptote (Δ) eventually becomes smaller than any distance that one may specify.*
If a curve (C) has the curve (Δ) as an asymptote, one says that (C) is asymptotic to (Δ). Similarly (Δ) is asymptotic to (C), so (C) and (Δ) are called asymptotic.
Essentially, a linear asymptote is a straight line, a line that a graph approaches, but does not become identical to.



1. **Horizontal asymptote :**
   the line y = b is a horizontal asymptote for (C_f) if
   $\lim_{x\to+\infty} f(x) = b$
   or $\lim_{x\to-\infty} f(x) = b$

2. **Vertical asymptote :**
   the line x = a is a Vertical asymptote for (C_f) if
   $\lim_{x\to a} f(x) = \pm\infty$
   or $\lim_{x\to a^+} f(x) = \pm\infty$
   or $\lim_{x\to a^-} f(x) = \pm\infty$



3. **Oblique asymptote :**
   the line y =a x+b is an oblique asymptote for (C_f) if
   $\lim_{x\to+\infty} [f(x) - (ax+b)] = 0$
   or $\lim_{x\to-\infty} [f(x) - (ax+b)] = 0$

   NB : if a = 0 then the asymptote is horizontal

4. **Position of (C_f) with respect to its asymptote** *(oblique or horizontal) :*

   If $\lim_{x\to\pm\infty} [f(x) - (ax+b)] = 0^+$ *then* (C_f) *is above* (Δ)

   If $\lim_{x\to\pm\infty} [f(x) - (ax+b)] = 0^-$ *then* (C_f) *is under* (Δ)

5. **Theorem :** If $\lim_{x\to\pm\infty} \frac{f(x)}{x} = a$ *then the line* $y = ax$ *is an asymptotic direction for* (C_f)

   If $\lim_{x\to\pm\infty} [f(x) - ax] = b$ *then the line* $y = ax + b$ *is an asymptote for* (C_f)

• To find an asymptote for an infinite branch (x -> ∞ and/or y -> ∞) then one must start by calculate the $\lim_{x\to\pm\infty} \frac{f(x)}{x}$.