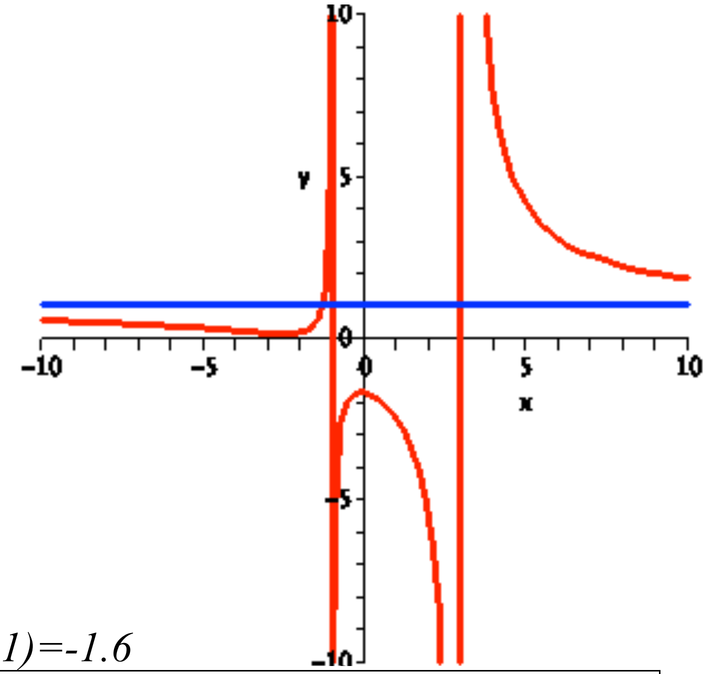
APPLICATIONS of the DERIVATIVES

Use the general formulas to compute the derivatives of the following functions, study their domain of existence, their sign and determine the variations on each interval (chart).



$$f(x)=\frac{x^2+4x+5}{x^2-2x-3}$$

$$f'(x)=-\frac{2(3x^2+8x+1)}{(x+1)(x-3)}$$

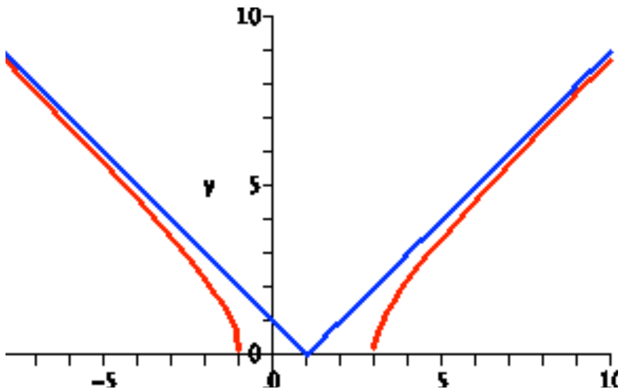
D_f =]-∞ ; -1[∪]-1 ; 3[∪]3 ; +∞[

Asymptotes : *x = -1 ; x = 3 ; y = 1*

$$f'(x)=0 \Leftrightarrow x=\frac{-4\pm\sqrt{13}}{3} \Leftrightarrow x \approx \text{-2.5 or -0.1}$$

Min : m = f(-2.5) = 0.1 ; Max : M = f(-0.1)=-1.6

x	- ∞ -2.5 -1 -0.1 3 + ∞
Sign [f'(x)]	— 0 ⊕ ‖ ⊕ 0 — ‖ —
Var° & lim. f_1	1^- ↘ 0.1 ↗ +∞ ‖ -∞ ↗ **-1.6** ↘ -∞ ‖ +∞ ↘ $\mathbf{1^+}$



$$f(x)=\sqrt{x^2-2x-3} \;\; ; \;\; D_f =]-\infty;-1] \cup [3;+\infty[$$

$$f'(x)=\frac{x-1}{\sqrt{x^2-2x-3}} \;\; ; \;\; Sgn[f'(x)] = Sgn[x-1]$$

$$\textit{Oblique Asymptote}: y=|x-1|$$

$$\because \lim_{x\to+\infty}[f(x)-(x-1)] = \lim_{x\to+\infty}\left[\frac{-4}{\sqrt{x^2-2x-3}+(x-1)}\right]=0^-$$

$$\lim_{x\to-\infty}[f(x)-(-x+1)] = \lim_{x\to-\infty}\left[\frac{-4}{\sqrt{x^2-2x-3}-(x+1)}\right]=0^-$$

x	- ∞ -1 1 3 + ∞
Sign [f_2'(x)]	— ‖ ////////////////////// ‖ ⊕
Var° & lim. f_1	+∞ ↘ **0** ////////////////////// **0** ↗ +∞