

Examples of Formal definition of Limits

N°	Notation	Definition	Illustration & Example
1	$\lim_{x \rightarrow a} f(x) = b$	$\forall \varepsilon > 0, \exists \alpha > 0,$ $ x - a < \alpha \Rightarrow f(x) - b < \varepsilon$ <hr/> <ul style="list-style-type: none"> $f(a)$ not necessarily defined. <hr/> <ul style="list-style-type: none"> Definition $\lim_{x \rightarrow a} f(x) = f(a)$ $\Leftrightarrow f \text{ is continuous @ } x = a$	$\lim_{x \rightarrow 2} \frac{x^3 - 8}{4x - 8} = \lim_{x \rightarrow 2} \left(\frac{1}{4}x^2 + \frac{1}{2}x + 1 \right) = 3$ <ul style="list-style-type: none"> $\lim_{x \rightarrow -4} \frac{x^3 - 8}{4x - 8} = f(-4) = 3$
2	$\lim_{x \rightarrow +\infty} f(x) = +\infty$	$\forall A > 0, \exists B > 0,$ $x > B \Rightarrow f(x) > A$ <hr/> <p>Ex : $\forall A > 0, \exists B \geq 2\sqrt{A},$ $x > B \geq 2\sqrt{A} \Rightarrow f(x) = x^2/4 > A$</p>	$\lim_{x \rightarrow +\infty} \frac{1}{4}x^2 = +\infty$
3	$\lim_{x \rightarrow +\infty} f(x) = b^+$	$\forall \varepsilon > 0, \exists B > 0,$ $x > B \Rightarrow b - \varepsilon < f(x) < b$ <hr/> <p><u>Definition :</u></p> <p>the line $y = b$ is a « horizontal » Asymptote to the graph of f</p>	$\lim_{x \rightarrow +\infty} \frac{3x + 4}{x + 1} = \lim_{x \rightarrow +\infty} 3 + \frac{1}{x + 1} = 3^+$
4	$\lim_{x \rightarrow a^+} f(x) = +\infty$	$\forall A > 0, \exists \alpha > 0,$ $a - \alpha < x < a \Rightarrow f(x) > A$ <hr/> <p><u>Definition :</u></p> <p>the line $x = a$ is a « vertical » Asymptote to the graph of f</p>	$\lim_{x \rightarrow -1^+} \frac{3x + 4}{x + 1} = +\infty$