

Let  $f$  be the function defined by  $f(x) = -\frac{1}{2}x + 2$  for  $x \geq 0$ .

Study of the sequence  $(u_n)$  defined by  $u_{n+1} = f(u_n) = -\frac{1}{2}u_n + 2$  ;  $n \geq 1$  and  $u_0 = 0$ .

1. Graph the function  $f$  on  $[0 ; +\infty [$  and draw the first terms of the sequence  $(u_n)$ .  
 Find the coordinates of the intersection of  $(C_f)$  with the first bisector ( $y=x$ )  
 Indicate from the graph whether or not the sequence is :
  - i. Monotonous (if yes how) :
  - ii. Bounded (if yes, what are the boundaries ?)
  - iii. Does-it seem to have a limit (if yes which one is it?)?

2. Let  $v_n = u_n - \frac{4}{3}$  for any  $n > 0$ .

Show that the new sequence  $(v_n)$  is a geometric sequence :

1. Find its first term and its reason.
2. Find the expression of  $v_n$  directly in function of  $n$ .
3. Deduct the limit of  $v_n$ .
4. Find the expression of  $u_n$  in function of  $v_n$
5. Find the limit of  $u_n$
6. Check the result on your graph.

