All that you ever wanted to know about Sequences without ever daring ask about them….

Definition : any list of Numbers written in a certain order is making a Numerical Sequence. We use a special notation to specify each term of the sequence in reference to it's rank in the list, this notation is generally U(n) or simply U_n. The index n shows the position of the number in the list. This index is a Natural number : 0, 1, 2 , 3, …, n-1, n, n+1,… The terms U_{n-1}, U_n and U_{n+1} are three following numbers in the sequence. U_0 represents the initial term of the sequence (rank 0 = 1st term).

Examples : *(1) Sequence of the numbers obtained by counting 3 by 3 from -5 :*
(-5, -2, 1, 4, 7, …,) the general term of this sequence is $U_n = -5 + 3.n$
The first term is $U_0 = -5$ *; the* 11^{th} *term is* $U_{10} = -5 + 3 \times 10 = 25$ *;* 100^{th} *term* $U_{99} = -5 + 3 \times 99 = 292$.

(2) Sequence of the numbers obtained by multiplying every previous number by 2 starting with 3 :
(3, 6, 12, 24, 48, …) The general term of that sequence is $Vn = 3.(2)^n$
First term : $V_0 = 3$ *;* 10^{th} *term :* $V_9 = 3.(2)^9 = 3 \times 512 = 1536$ *;* $V_{20} = 3.(2)^{20} = 3 \times 1024^2 = 3\ 145\ 728$.

(3) Sequence of the decimals digits of the number π : (3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9 …..) In this sequence the 10th term (rank n = 9) is 5, but there is no simple formula to find it …

(4) Fibonacci sequence : (1, 1 , 2, 3, 5, 8, 13, 21, 34, …) in this sequence every term is the sum of the two previous ones. We can get as many terms as we want but we cannot get the value of the 100^{th} term without calculating the 99 terms before it, We have : $U_{n+1} = U_n + U_{n-1}$, But the formula that would provide the direct value is complicated (see Binet formula on the website)

(5) Sequencce of the squares of the integers : (0, 1, 4, 9, 16, 25, 36, …) It's easy to find that $U_n = n^2$ and that $U_{100} = 100^2 = 10\ 000$.

The type (1) sequences are named **Arithmetic Sequences,**
The type (2) sequences are named **Geometric Sequences,**
The sequences (3) , (4) , (5) are neither arithmetic or geometric.

ARITHMETIC Sequence	**GEOMETRIC Sequence**
Definitions 1	
Every term is obtained by adding the same number **r** to the previous one (**r** = « reason »)	Every term is obtained by multiplying each term by the same number **q** (**q** = « quotient»)
$U_{n+1} = U_n + r$	$V_{n+1} = q.V_n$
Definitions 2	
The **difference** of two following terms is **constant**	The **quotient** of two following terms is **constant**
$U_{n+1} - U_n = r$	$\frac{V_{n+1}}{V_n} = q$
General Formulas	
$U_n = a + n.r$	$V_n = a.q^n$
1st term : $U_0 = a$; reason = **r** (*ratio* = difference)	1st term : $V_0 = a$; reason = **q** (quotient = *ratio*)
Characteristic property # 1	
Every term is the arithmetic mean of the terms which are équidistants from it. $U_n = \frac{U_{n-p} + U_{n+p}}{2}$	Every term is the quadratic mean of the terms which are équidistants from it. $U_n = \sqrt{U_{n-p}.U_{n+p}}$
Characteristic property # 2	
Variation of **Linear** type	Variation of **Exponential** type