

All that you ever wanted to know about Sequences without ever daring ask about them....

Definition : any list of Numbers written in a certain order is making a Numerical Sequence. We use a special notation to specify each term of the sequence in reference to it's rank in the list, this notation is generally $U(n)$ or simply U_n . The index n shows the position of the number in the list. This index is a Natural number: $0, 1, 2, 3, \dots, n-1, n, n+1, \dots$. The terms U_{n-1}, U_n and U_{n+1} are three following numbers in the sequence. U_0 represents the initial term of the sequence (rank $0 = 1$ st term).

Examples : (1) Sequence of the numbers obtained by counting 3 by 3 from -5 :

(-5, -2, 1, 4, 7, ...) the general term of this sequence is $U_n = -5 + 3n$

The first term is $U_0 = -5$; the 11th term is $U_{10} = -5 + 3 \times 10 = 25$; 100th term $U_{99} = -5 + 3 \times 99 = 292$.

(2) Sequence of the numbers obtained by multiplying every previous number by 2 starting with 3 :

(3, 6, 12, 24, 48, ...) The general term of that sequence is $V_n = 3 \cdot (2)^n$

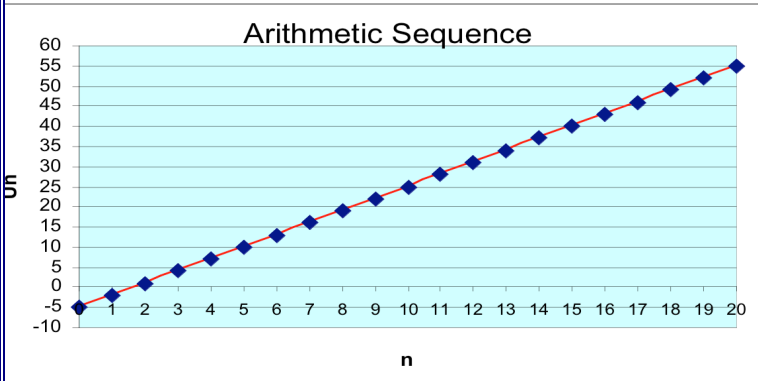
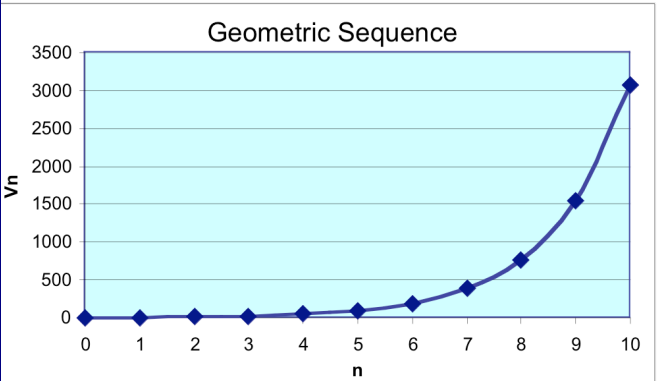
First term : $V_0 = 3$; 10th term : $V_9 = 3 \cdot (2)^9 = 3 \times 512 = 1536$; $V_{20} = 3 \cdot (2)^{20} = 3 \times 1024^2 = 3 \times 145 \ 728$.

(3) Sequence of the decimals digits of the number π : (3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9 ...) In this sequence the 10th term (rank $n = 9$) is 5, but there is no simple formula to find it ...

(4) Fibonacci sequence : (1, 1, 2, 3, 5, 8, 13, 21, 34, ...) in this sequence every term is the sum of the two previous ones. We can get as many terms as we want but we cannot get the value of the 100th term without calculating the 99 terms before it, We have : $U_{n+1} = U_n + U_{n-1}$, But the formula that would provide the direct value is complicated (see Binet formula on the website)

(5) Sequence of the squares of the integers : (0, 1, 4, 9, 16, 25, 36, ...) It's easy to find that $U_n = n^2$ and that $U_{100} = 100^2 = 10 \ 000$.

The type (1) sequences are named **Arithmetic Sequences**,
 The type (2) sequences are named **Geometric Sequences**,
 The sequences (3) , (4) , (5) are neither arithmetic or geometric.

ARITHMETIC Sequence	GEOMETRIC Sequence
Definitions 1	
Every term is obtained by adding the same number r to the previous one (r = « reason ») $U_{n+1} = U_n + r$	Every term is obtained by multiplying each term by the same number q (q = « quotient») $V_{n+1} = q \cdot V_n$
Definitions 2	
The difference of two following terms is constant $U_{n+1} - U_n = r$	The quotient of two following terms is constant $\frac{V_{n+1}}{V_n} = q$
General Formulas	
$U_n = a + n \cdot r$ 1 st term : $U_0 = a$; reason = r (ratio = difference)	$V_n = a \cdot q^n$ 1 st term : $V_0 = a$; reason = q (quotient = ratio)
Characteristic property # 1	
Every term is the arithmetic mean of the terms which are équidistants from it. $U_n = \frac{U_{n-p} + U_{n+p}}{2}$	Every term is the quadratic mean of the terms which are équidistants from it. $U_n = \sqrt{U_{n-p} \cdot U_{n+p}}$
Characteristic property # 2	
Variation of Linear type	Variation of Exponential type
 <p style="text-align: center;">Arithmetic Sequence</p>	 <p style="text-align: center;">Geometric Sequence</p>