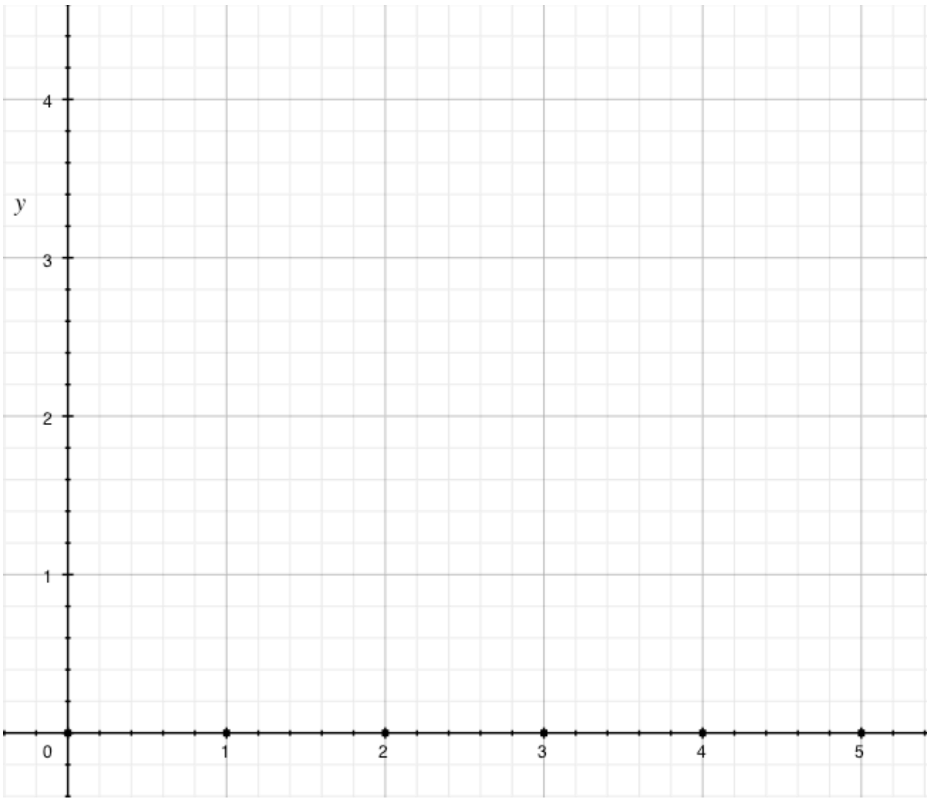
Sequences defined by recurrence (3)

Problem I : Let *f* be the function defined by $f(x) = \dfrac{4x-2}{x+1}$ for x ≥ 0.

Study of the sequence (v_n) defined by $v_{n+1} = f(v_n) = \dfrac{4v_n - 2}{v_n + 1}$; n ≥1 and $v_0 = 4$.

1. Graph the function f on [0 ; +∞ [and draw the first terms of the sequence (v_n).
   Find the coordinates of the intersection of (Cf) with the first bisector ($y = x$)
   Indicate from the graph whether or not the sequence is :
   i. Monotonous (if yes how) :
   ii. Bounded (*if yes, what are the boundaries ?)*
   iii. Does-it seam to have a limit, *if yes which one is it* ?
2. Same questions if $v_0 = 0$, or $v_0 = 1$, or $v_0 = 1.5$ or $v_0 = 2$.
3. Let $w_n = \dfrac{v_n - 2}{v_n - 1}$ *for any n>0.*

   Show that the new sequence (w_n) is a **geometric** sequence :
   1. Find its first term and its reason.
   2. Find the expression of w_n directly in function of n.
   3. Deduct the limit of w_n.
   4. Find the expression of v_n in function *of* w_n
   5. Find the limit of v_n
   6. Check the *result on your graph.*



Problem II : Let f be the function defined by $f(x) = -\frac{1}{2}x + 2$ for x ≥ 0.

Study of the sequence (u_n) defined by $u_{n+1} = f(u_n) = -\frac{1}{2}u_n + 2$; n ≥1 and $u_0 = 0$.

1. Graph the function f on [0 ; +∞ [and draw the first terms of the sequence (u_n).
   Find the coordinates of the intersection of (Cf) with the first bisector (y=x)
   Indicate from the graph whether or not the sequence is :
   i. Monotonous (if yes how) :
   ii. Bounded (*if yes, what are the boundaries ?)*
   iii. Does-it seam to have a limit (*if yes which one is it?*)?
   iv. Is this sequence Arithmetic or Geometric or neither ?
2. Let $v_n = u_n - \frac{4}{3}$ *for any n>0.*
   Show that the new sequence (v_n) is a **geometric** sequence :
   1. Find its first term and its reason.
   2. Find the expression of v_n directly in function of n.
   3. Deduct the limit of v_n.
   4. Find the expression of u_n in function *of* v_n
   5. Find the limit of u_n
   6. Check the *result on your graph.*