N° 82 p. 141 ① $f(t)=\frac{t}{(t^2-1)^2}$ $t\neq -1 ; t\neq 1$ 2/2 **26/01/08**

$F(t)=\left(-\frac{1}{2}\right)\frac{1}{t^2-1}$ car $f(t)= -\frac{1}{2}\left(\frac{-2t}{(t^2-1)^2}\right)$ Forme : $k\left(\frac{-u'}{u^2}\right)=\left(\frac{k}{u}\right)'$

② $t\neq 0 ; t\neq 1 ; t\neq -1$ $g(t)=\frac{1}{t(t^2-1)}=\frac{a}{t}+\frac{b}{t-1}+\frac{c}{t+1}$

Par identification : $1=a(t^2-1)+bt(t+1)+ct(t-1)=(a+b+c)t^2+(b-c)t-a$

on trouve $a=-1$; $a+b+c=0$; $b-c=0 \Rightarrow b=c=\frac{1}{2}$.

d'où $g(t)=-\frac{1}{t}+\frac{1}{2}\times\frac{1}{t-1}+\frac{1}{2}\frac{1}{t+1}$.

$\Rightarrow G(t)=-\ln|t|+\frac{1}{2}\ln|t-1|+\frac{1}{2}\ln|t+1| = \ln\frac{\sqrt{t^2-1}}{t}$ $(t>1)$. $(+k)$

③ $J(x)=\int_2^x \frac{t\ln t}{(t^2-1)^2}\,dt=\int_2^x \underbrace{\ln t}_{u}\cdot\underbrace{\frac{t}{t^2-1}}_{v'}\,dt = \underbrace{\left[-\frac{\ln t}{2(t^2-1)}\right]_2^x}_{[u.v]_2^x}+\underbrace{\frac{1}{2}\int_2^x g(t)\,dt}_{-\int_2^x u'\times v}$

$u'=\frac{1}{t}$; $v=-\frac{1}{2}\frac{1}{t^2-1}$

$\Rightarrow J(x)=\frac{\ln 2}{6}-\frac{\ln x}{2(x^2-1)}+\frac{1}{2}\left[\ln\frac{\sqrt{t^2-1}}{t}\right]_2^x = \frac{\ln 2}{6}-\frac{1}{2}\frac{\ln x}{x^2-1}+\frac{1}{4}\ln(x^2-1) - \frac{1}{2}\ln x-\frac{1}{4}\ln 3+\frac{\ln 2}{2}$

$\Rightarrow J(x)=\left[-\frac{1}{2}\right]\left[\frac{1}{x^2-1}+1\right]\ln x+\frac{1}{4}\ln(x^2-1)+\frac{2}{3}\ln 2-\frac{1}{4}\ln 3$.

$h(x)=-\frac{x^2}{2x^2-2}$; $A=\frac{1}{4}$; $B=\frac{2}{3}\ln 2-\frac{1}{4}\ln 3$. ■

N° 83 p. 142 $K=\int_{\sqrt{2}}^2\frac{dx}{\sqrt{x^2-1}}$; $f(x)=\ln\left(x+\sqrt{x^2-1}\right)$ $x>1$

$f'(x)=\frac{1+\frac{x}{\sqrt{x^2-1}}}{x+\sqrt{x^2-1}}=\frac{\frac{x+\sqrt{x^2-1}}{\sqrt{x^2-1}}}{x+\sqrt{x^2-1}}=\frac{1}{\sqrt{x^2-1}} \Rightarrow K=\left[\ln\left(x+\sqrt{x^2-1}\right)\right]_{\sqrt{2}}^2$

$\Rightarrow K=\ln(2+\sqrt{3})-\ln(\sqrt{2}+1)=\boxed{\ln\frac{2+\sqrt{3}}{1+\sqrt{2}}}$

$J=\int_{\sqrt{2}}^2\sqrt{x^2-1}\,dx \Rightarrow J+K=\int_{\sqrt{2}}^2\left(\sqrt{x^2-1}+\frac{1}{\sqrt{x^2-1}}\right)dx=\int_{\sqrt{2}}^2\frac{x^2}{\sqrt{x^2-1}}\,dx$

$\int_{\sqrt{2}}^2 x\cdot\frac{2x}{2\sqrt{x^2-1}}\,dx=\left[x\sqrt{x^2-1}\right]_{\sqrt{2}}^2-\int_{\sqrt{2}}^2\sqrt{x^2-1}\,dx=(2\sqrt{3}-\sqrt{2})-J$

$x \rightarrow 1$; $\frac{2x}{2\sqrt{x^2-1}} \leftarrow \sqrt{x^2-1}$

Donc $J+K=2\sqrt{3}-\sqrt{2}-J \Rightarrow 2J=2\sqrt{3}-\sqrt{2}-K$

$\Rightarrow J=\frac{1}{2}\left[(2\sqrt{3}-\sqrt{2})-\ln\frac{2+\sqrt{3}}{1+\sqrt{2}}\right]$. ■