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①/ 2

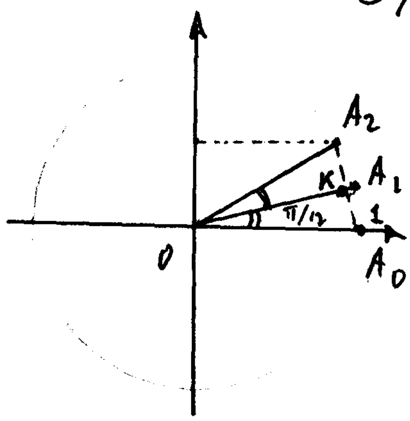
$z_0 = 1 \longleftrightarrow A_0$

$z_1 = e^{i\frac{\pi}{12}} \longleftrightarrow A_1$

$z_2 = e^{i\frac{\pi}{6}} \longleftrightarrow A_2$

K milieu de $[A_0A_2] \iff \vec{OK} = \frac{1}{2}(\vec{OA_0} + \vec{OA_2})$

$\iff z_K = \frac{1}{2}(z_0 + z_2) = \frac{1}{2}(1 + e^{i\frac{\pi}{6}})$



1) $e^{i\frac{\pi}{6}} = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2} + i\cdot\frac{1}{2} \Rightarrow \boxed{z_K = \frac{2+\sqrt{3}}{4} + i\cdot\frac{1}{4}}$

2) Dans le triangle OKA_2 rectangle en K on a $OK = \cos\frac{\pi}{12}$

et $(\vec{OA_0}; \vec{OK}) = (\vec{OA_0}; \vec{OA_1}) = \operatorname{Arg}(z_1) = \frac{\pi}{12}$

$\Rightarrow \boxed{z_K = \left(\cos\frac{\pi}{12}\right) e^{i\frac{\pi}{12}}} = \left(\cos\frac{\pi}{12}\right)\left[\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right] = \cos^2\frac{\pi}{12} + i\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}$

3°) $(\sqrt{6}+\sqrt{2})^2 = 6 + 2\sqrt{12} + 2 = 8 + 4\sqrt{3} \Rightarrow \sqrt{6}+\sqrt{2} = \sqrt{8+4\sqrt{3}}$

$z_K = \cos^2\frac{\pi}{12} + i\,\frac{1}{2}\sin\frac{\pi}{6} = \frac{2+\sqrt{3}}{4} + i\,\frac{1}{4} \iff \begin{cases} \cos^2\frac{\pi}{12} = \frac{2+\sqrt{3}}{4} & (1) \\ \frac{1}{2}\sin\frac{\pi}{6} = \frac{1}{4} \end{cases}$

$\Rightarrow \cos\frac{\pi}{12} = \frac{\sqrt{2+\sqrt{3}}}{2} = \frac{2\sqrt{2+\sqrt{3}}}{4} = \frac{\sqrt{8+4\sqrt{3}}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$ CQFD.

et $\sin\frac{\pi}{12} = \sqrt{1-\cos^2\frac{\pi}{12}} = \sqrt{1 - \frac{8+4\sqrt{3}}{16}} = \sqrt{\frac{8-4\sqrt{3}}{16}} = \frac{\sqrt{8-4\sqrt{3}}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$ CQFD

II. N°107 p. 197 $\quad \omega = e^{2i\frac{\pi}{5}}$

1°) $\omega^5 = \left(e^{2i\frac{\pi}{5}}\right)^5 = e^{2i\pi} = 1$; $1 + \omega + \omega^2 + \omega^3 + \omega^4 = \frac{1-\omega^5}{1-\omega} = 0$.

2°) $u + v = (\omega + \omega^4) + (\omega^2 + \omega^3) = -1$

$uv = (\omega + \omega^4)(\omega^2 + \omega^3) = \omega^3 + \omega^6 + \omega^4 + \omega^7 = \omega^3 + \omega + \omega^4 + \omega^2 = -1$

Donc u et v sont sol de l'équation $Z^2 - SZ + P = 0$

avec $S = u + v = -1$ et $P = uv = -1$ $\qquad Z^2 + Z - 1 = 0$ dont les racines

sont $\frac{-1+\sqrt{5}}{2}$ et $\frac{-1-\sqrt{5}}{2}$

3°) $u = \omega + \omega^4$ mais $\omega^4 = \frac{\omega^5}{\omega} = \frac{1}{\omega} = \bar{\omega}$ car $\omega\bar{\omega} = |\omega|^2 = 1$.

Donc $u = \omega + \bar{\omega} = 2\operatorname{Re}[\omega] = 2\left[\cos\frac{2\pi}{5}\right] > 0$.

$\Rightarrow u = \frac{-1+\sqrt{5}}{2} = 2\cos\frac{2\pi}{5} \iff \boxed{\cos\frac{2\pi}{5} = \frac{-1+\sqrt{5}}{4}.}$

(on peut vérifier ce résultat avec une calculatrice).

- $\sin\frac{2\pi}{5} = \sqrt{1-\cos^2\frac{2\pi}{5}} = \sqrt{1-\left(\frac{-1+\sqrt{5}}{4}\right)^2} = \sqrt{\frac{16-(6-2\sqrt{5})}{16}} = \frac{\sqrt{10+2\sqrt{5}}}{4}$

TSVP →