I – Calcul d'Intégrales.

1°) $g(x) = \dfrac{1}{x(x^2-1)}$ $\boxed{x > 1}$.

a) $g(x) = \dfrac{a}{x} + \dfrac{b}{x+1} + \dfrac{c}{x-1} = \dfrac{a(x^2-1) + bx(x-1) + c(x+1)x}{x(x^2-1)}$.

$\Longleftrightarrow (a+b+c)x^2 + (c-b)x + (-a) = 1$ pour tout $x > 1$

$\Longleftrightarrow \left\{\begin{array}{l} a+b+c = 0 \\ c-b = 0 \\ a = -1 \end{array}\right\} \Longleftrightarrow \left\{\begin{array}{l} a = -1 \\ b = c = \frac{1}{2} \end{array}\right\} \Rightarrow \boxed{g(x) = -\dfrac{1}{x} + \dfrac{1}{2(x-1)} + \dfrac{1}{2(x+1)}}$

b) $G(x) = -\ln x + \frac{1}{2}\ln(x-1) + \frac{1}{2}\ln(x+1) + k$. $(x>1)$ Primitive de g.

2°) $f(x) = \dfrac{2x}{(x^2-1)^2}$ a pour primitive $F(x) = -\dfrac{1}{x^2-1}$ $\left(\text{forme } \dfrac{u'}{u^2} = \left(-\dfrac{1}{u}\right)'\right)$

3°) $I = \displaystyle\int_2^3 \frac{2x}{(x^2-1)^2}\ln x\,dx = \left[-\frac{\ln x}{x^2-1}\right]_2^3 - \int_2^3 \frac{-1}{x(x^2-1)}dx$

$\uparrow -\dfrac{1}{(x^2-1)}$ $\quad \downarrow \dfrac{1}{x}$

$= \left[\dfrac{-\ln x}{x^2-1}\right]_2^3 + \left[G(x)\right]_2^3$

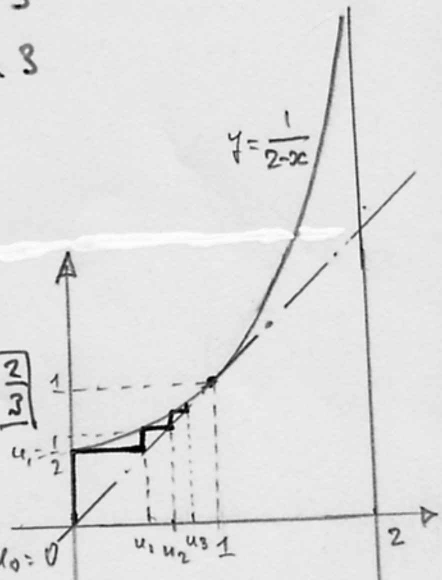
$\Longleftrightarrow I = \left[\dfrac{-\ln x}{x^2-1} - \ln x + \dfrac{\ln}{2}(x-1) + \dfrac{1}{2}\ln(x+1)\right]_2^3$

$\Longleftrightarrow I = \left(-\dfrac{\ln 3}{8} - \ln 3 + \dfrac{1}{2}\ln 2 + \dfrac{1}{2}\ln 4\right) - \left(\dfrac{-\ln 2}{3} - \ln 2 + \dfrac{1}{2}\ln 1 + \dfrac{1}{2}\ln 3\right)$

$\Longleftrightarrow I = \left(\dfrac{1}{2} + 1 + 1 + \dfrac{1}{3}\right)\ln 2 - \left(\dfrac{1}{8} + 1 + \dfrac{1}{2}\right)\ln 3$

$\Longleftrightarrow \boxed{I = \dfrac{17}{6}\ln 2 - \dfrac{13}{8}\ln 3} \simeq 2,6 > 0$

III – Etude d'une SUITE Numérique



1°) $\left\{\begin{array}{l} u_{n+1} = \dfrac{1}{2-u_n} \\ u_0 = 0 \end{array}\right.$ $\Big|$ $u_1 = \dfrac{1}{2-u_0} = \boxed{\dfrac{1}{2}}$ $\quad u_2 = \dfrac{1}{2-u_1} = \dfrac{1}{2-\frac{1}{2}} = \boxed{\dfrac{2}{3}}$

$u_3 = \dfrac{1}{2-u_2} = \dfrac{1}{2-\frac{2}{3}} = \boxed{\dfrac{3}{4}}$.

b) $w_n = \dfrac{n}{n+1} \Rightarrow w_0 = 0 = u_0$, $w_1 = \frac{1}{2} = u_1$ $\Big|$ $w_2 = \frac{2}{3} = u_2$, $w_3 = \frac{3}{4} = u_3$ $\Big|$ D'après la fig. $u_0 = 0$, $\lim u_n = 1$

$u_{n+1} = f(u_n)$ avec $f(x) = \dfrac{1}{2-x}$

$f'(x) = \dfrac{1}{(2-x)^2} > 0$ $f \nearrow [0,2[$

c) Soit (H_n) la relation $\boxed{u_n = w_n}$. INIT : $(H_0) \Longleftrightarrow u_0 = w_0$ VRAI

$(H_n) \Rightarrow (H_{n+1})$? en effet $u_{n+1} = \dfrac{1}{2-u_n} = \dfrac{1}{2-w_n} = \dfrac{1}{2-\frac{n}{n+1}} = \dfrac{n+1}{n+2} = w_{n+1}$ CQFD

Donc (H_n) Vrai pour tout $n \in \mathbb{N}$.

2°) $v_n = \ln\left(\dfrac{n}{n+1}\right) \longrightarrow v_1 + v_2 + v_3 = \ln\left(\dfrac{1}{2}\right) + \ln\left(\dfrac{2}{3}\right) + \ln\left(\dfrac{3}{4}\right) = \ln\left(\dfrac{1}{2} \times \dfrac{2}{3} \times \dfrac{3}{4}\right) = \ln\left(\dfrac{1}{4}\right)$

$S_n = v_1 + v_2 + \dots + v_n = \ln\frac{1}{2} + \dots + \ln\dfrac{n}{n+1} = \ln\left(\dfrac{1}{2} \times \dfrac{2}{3} \times \dots \times \dfrac{n}{n+1}\right) = \ln\left(\dfrac{1}{n+1}\right) \longrightarrow \boxed{-\infty}$