8-4 SOLVING QUADRATIC EQUATIONS BY USING THE QUADRATIC FORMULA

Any quadratic equation may be solved by completing the square. When the general quadratic equation, $ax^2 + bx + c = 0$, $a \neq 0$, is solved in this way, the result is the **quadratic formula**.

$$ax^2 + bx + c = 0$$
$$ax^2 + bx = -c$$
$$D_a \quad x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Complete the square for the left member and add the same number to the right member.

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$
$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$$
$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$
$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

In developing this for better students you may wish to write the form $x^2 - a^2 = 0$. Then factor, $(x + a)(x - a) = 0$ and use the fact that one or the other factor must be zero.

$$x + \frac{b}{2a} = \frac{\sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x + \frac{b}{2a} = -\frac{\sqrt{b^2 - 4ac}}{2a}$$
$$x = -\frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a} \quad\quad x = -\frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad\quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

THE QUADRATIC FORMULA

For the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, the solutions are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Remind students that the right member represents two numbers. Ask students to identify them.

To use the quadratic formula for solving a quadratic equation, replace a, b, and c in the formula by the corresponding coefficients from the equation.

EXAMPLE 1 Solve $x^2 + 2x - 3 = 0$. Use the quadratic formula.

SOLUTION Comparing the equations

$$ax^2 + bx + c = 0 \text{ and}$$
$$x^2 + 2x - 3 = 0,$$

we see that $a = 1$, $b = 2$, and $c = -3$.

Replacing a, b, and c by these numbers in the quadratic formula gives $x = \dfrac{-2 \pm \sqrt{2^2 - 4(1)(-3)}}{2(1)}$

$$= \frac{-2 \pm \sqrt{16}}{2}$$
$$= \frac{-2 \pm 4}{2}$$
$$= -1 \pm 2$$

In solving quadratic equations, have students write the formula once, if at all, on their paper. In the solution itself, they should utilize the format of the formula to structure the expression for x without having to write the formula each time.

$$x = -1 + 2 \quad \text{or} \quad x = -1 - 2$$
$$x = 1 \quad\quad x = -3$$

CHECK The check is left for the student.

EXAMPLE 2 Solve $3x^2 - 4x - 2 = 0$.

SOLUTION $3x^2 - 4x - 2 = 0$, $a = 3$, $b = -4$, $c = -2$

By the formula, $x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-2)}}{2 \cdot 3}$

$$= \frac{4 \pm \sqrt{40}}{6}$$
$$= \frac{4 \pm 2\sqrt{10}}{6}$$
$$= \frac{2 \pm \sqrt{10}}{3}$$

For equations having irrational-number solutions, the student may approximate the solution with a decimal. Then, checking the approximate solution would provide good practice in using a calculator.

$$x = \frac{2}{3} + \frac{\sqrt{10}}{3} \quad \text{or} \quad x = \frac{2}{3} - \frac{\sqrt{10}}{3}$$

CHECK

$$3x^2 - 4x - 2 = 0$$
$$3\left(\frac{2}{3} + \frac{\sqrt{10}}{3}\right)^2 - 4\left(\frac{2}{3} + \frac{\sqrt{10}}{3}\right) - 2 \stackrel{?}{=} 0$$
$$3\left(\frac{4}{9} + \frac{4\sqrt{10}}{9} + \frac{10}{9}\right) - 4\left(\frac{2}{3} + \frac{\sqrt{10}}{3}\right) - 2 \stackrel{?}{=} 0$$
$$\frac{4}{3} + \frac{4\sqrt{10}}{3} + \frac{10}{3} - \frac{8}{3} - \frac{4\sqrt{10}}{3} - 2 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

The check for $x = \frac{2}{3} - \frac{\sqrt{10}}{3}$ is left for the student.